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Dynamics of low-energy nuclear forces for electromagnetic and weak reactions with the deuteron in the Nambu-Jona-Lasinio model of light nuclei

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Abstract. A dynamics of low-energy nuclear forces is investigated for low-energy electromagnetic and weak nuclear reactions with the deuteron in the Nambu-Jona-Lasinio model of light nuclei by example of the neutron-proton radiative capture (M1-capture) n + p → D + γ, the photomagnetic disintegration of the deuteron γ + D → n + p and weak reactions of astrophysical interest. These are the solar proton burning p + p → D + e^+ + ν_e, the pep-process p + e^- + p → D + ν_e and the neutrino and antineutrino disintegration of the deuteron caused by charged ν_e + D → e^- + p + p, $\bar{\nu}_e$ + D → e^+ + n + n and neutral $\nu_e(\bar{\nu}_e)$ + D → $\nu_e(\bar{\nu}_e)$ + n + p weak currents.

PACS. 11.10.Ef Lagrangian and Hamiltonian approach – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 14.20.Dh Protons and neutrons – 21.30.Fe Forces in hadronic systems and effective interactions

1 Introduction

Recently [1] we have developed the Nambu-Jona-Lasinio model of light nuclei [2], or differently the nuclear Nambu-Jona-Lasinio (NNJL) model, invented for the description of low-energy nuclear forces at the quantum field-theoretic level. We have shown that the NNJL model is fully motivated by QCD [1]. The deuteron appears in *the nuclear phase of QCD* as a neutron-proton collective excitation, a Cooper np-pair, caused by a phenomenological local four-nucleon interaction. Strong low-energy interactions of the deuteron coupled to itself and other particles are described in terms of one-nucleon loop exchanges. The one-nucleon loop exchanges allow to transfer nuclear flavours from an initial to a final nuclear state by a minimal way and to take into account contributions of nucleon-loop anomalies determined completely by one-nucleon loop diagrams. The dominance of contributions of nucleon-loop anomalies is justified in the large N_C approach to the description of non-perturbative QCD with $SU(N_C)$ gauge group at $N_C \to \infty$, where N_C is the number of quark colours.

In this paper we apply the NNJL model to the description of low-energy nuclear forces for electromagnetic and weak reactions with the deuteron by example of the evaluation of the cross-sections for the neutron-proton radiative capture n + p → D + γ for thermal neutrons caused by the ${}^1S_0 \to {}^3S_1$ transition (the M1-transition), the photomagnetic disintegration of the deuteron γ + D → n + p and weak reactions of astrophysical interest: 1) the solar proton burning p + p → D + e^+ + ν_e, 2) the pep-process p + e^- + p → D + ν_e and 3) the reactions of neutrino and antineutrino disintegration of the deuteron caused by charged ν_e + D → e^- + p + p, $\bar{\nu}_e$ + D → e^+ + n + n and neutral $\nu_e(\bar{\nu}_e)$ + D → $\nu_e(\bar{\nu}_e)$ + n + p weak currents.

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1.1 Low-energy electromagnetic nuclear reactions with the deuteron

It is well known that the reaction of the neutron-proton radiative capture n + p → D + γ for thermal neutrons plays an important role for the primordial nucleosynthesis in the Big-Bang model [3]. Indeed, the deuterons produced via the neutron-proton radiative capture n + p → D + γ burn to ^{4}He through the reactions D + p → ^{3}He + γ and ^{3}He + n → ^{4}He + γ. Therefore, the correct description of the neutron-proton radiative capture n + p → D + γ is essential for the theoretical prediction of the amount of the matter in the Universe.

The reaction of the photomagnetic disintegration of the deuteron γ + D → n + p is related to the neutron-proton radiative capture n + p → D + γ via time-reversal invariance of strong and electromagnetic forces. The investigation of this reaction is conceived to get an additional check of our result for the M1-capture related to the analysis of the energy dependence of the cross-section calculated in the NNJL model at energies far from threshold.

The cross-section $\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}})$ for the neutron-proton radiative capture has been measured for thermal neutrons at the laboratory kinetic energy $T_{\mathrm{n}} = 0.0252\,\mathrm{eV}$ that corresponds to the laboratory velocity $v_{\mathrm{n}}/c = 7.34 \times 10^{-6}$ (the absolute value is $v_{\mathrm{n}} = 2.2 \times 10^5\,\mathrm{cm/s}$) [4]:

$$\sigma(\mathrm{np} \to \mathrm{D}\gamma)_{\exp}(T_{\mathrm{n}}) = (334.2 \pm 0.5)\,\mathrm{mb}. \tag{1.1}$$

For the first time the cross-section for the neutron-proton radiative capture has been calculated by Austern [5] in the Potential model approach (PMA):

$$\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}}) = (303 \pm 4)\,\mathrm{mb}. \tag{1.2}$$

The observed discrepancy about 10% has been then explained by Riska and Brown [6] in terms of the contributions of the exchange currents and the $\Delta(1232)$ resonance.

Recently the evaluation of the cross-section for the neutron-proton radiative capture $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$ has been carried out by using Chiral perturbation theory in the framework of the Effective Field Theory (EFT) approach [7–10] formulated by Weinberg within Effective Chiral Lagrangian description of nuclear forces [11] (see also refs. [12]). The theoretical results obtained in refs. [7–10] for the cross-section for the neutron-proton radiative capture are rather contradictory. Indeed, in ref. [7] the experimental value of the cross-section for the M1-capture has been reproduced without free parameters, the definition of which demands a fit of experimental data, with an accuracy better than 1%, $\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}}) = (334 \pm 3)\,\mathrm{mb}$. In turn, in the more recent publications [8–10] the predictions are not so much optimistic. Indeed, in ref. [8] there has been obtained the value $\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}}) = 297.2\,\mathrm{mb}$ which is about 11% less compared with the experimental one eq. (1.1). As has been stated in ref. [8] for the evaluation of the correct value of the cross-section for the M1-capture within the EFT one needs to add a free parameter undefined in the approach. This parameter should be fixed from the fit of the experimental value eq. (1.1). This program has been realized in ref. [9]. Then, in refs. [10] the neutron-proton radiative capture has been calculated for the center of mass energies of the np pair up to $T_{\mathrm{np}} \le 1\,\mathrm{MeV}$, $T_{\mathrm{n}} = 2\,T_{\mathrm{np}}$, by including the contribution of the E1-transition in addition to the M1. This has added new free parameters with respect to that introduced in refs. [8,9], where only the M1-transition has been taken into account.

In the NNJL model we calculate the cross-section for the M1-capture both in the tree-meson approximation and by including the contributions of chiral one-meson loop corrections and the $\Delta(1232)$ resonance. For the evaluation of chiral one-meson loop corrections we apply chiral perturbation theory at the quark level $(\mathrm{CHPT})_{\mathrm{q}}$ [13] developed within the extended Nambu-Jona-Lasinio (ENJL) model with a linear realization of chiral $U(3) \times U(3)$ symmetry.

In our consideration the $\Delta(1232)$ resonance is the Rarita-Schwinger field [14] $\Delta^a_\mu(x)$, the isotopical index a runs over $a = 1, 2, 3$, having the following free Lagrangian [15,16]:

$$\begin{aligned}\mathcal{L}^{\Delta}_{\mathrm{kin}}(x) &= \bar{\Delta}^a_\mu(x)[-(i\gamma^\alpha\partial_\alpha - M_\Delta)\,g^{\mu\nu} \\ &\quad + \frac{1}{4}\gamma^\mu\gamma^\beta(i\gamma^\alpha\partial_\alpha - M_\Delta)\gamma_\beta\gamma^\nu]\Delta^a_\nu(x),\end{aligned} \tag{1.3}$$

where $M_\Delta = 1232\,\mathrm{MeV}$ is the mass of the Δ resonance field $\Delta^a_\mu(x)$. In terms of the eigenstates of the electric charge operator the fields $\Delta^a_\mu(x)$ are given by [15,16]

$$\begin{aligned}\Delta^1_\mu(x) &= \tfrac{1}{\sqrt{2}}\begin{pmatrix}\Delta^{++}_\mu(x) - \Delta^0_\mu(x)/\sqrt{3} \\ \Delta^+_\mu(x)/\sqrt{3} - \Delta^-_\mu(x)\end{pmatrix}, \\ \Delta^2_\mu(x) &= \tfrac{i}{\sqrt{2}}\begin{pmatrix}\Delta^{++}_\mu(x) + \Delta^0_\mu(x)/\sqrt{3} \\ \Delta^+_\mu(x)/\sqrt{3} + \Delta^-_\mu(x)\end{pmatrix}, \\ \Delta^3_\mu(x) &= -\sqrt{\tfrac{2}{3}}\begin{pmatrix}\Delta^+_\mu(x) \\ \Delta^0_\mu(x)\end{pmatrix}.\end{aligned} \tag{1.4}$$

The fields $\Delta^a_\mu(x)$ obey the subsidiary constraints: $\partial^\mu\Delta^a_\mu(x) = \gamma^\mu\Delta^a_\mu(x) = 0$ [14–16]. The Green function of the free Δ–field is determined by

$$\langle 0|\mathrm{T}(\Delta_\mu(x_1)\bar{\Delta}_\nu(x_2))|0\rangle = -iS_{\mu\nu}(x_1 - x_2). \tag{1.5}$$

In the momentum representation $S_{\mu\nu}(x)$ reads [15–17]:

$$\begin{aligned}S_{\mu\nu}(p) &= \frac{1}{M_\Delta - \hat{p}} \\ &\times\left(-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{M_\Delta} + \frac{2}{3}\frac{p_\mu p_\nu}{M^2_\Delta}\right).\end{aligned} \tag{1.6}$$

The most general form of the $\pi\mathrm{N}\Delta$-interaction compatible with the requirements of chiral symmetry reads [18]

$$\begin{aligned}\mathcal{L}_{\pi\mathrm{N}\Delta}(x) &= \frac{g_{\pi\mathrm{N}\Delta}}{2M_{\mathrm{N}}}\bar{\Delta}^a_\omega(x)\Theta^{\omega\varphi}N(x)\partial_\varphi\pi^a(x) + \mathrm{h.c.} \\ &= \frac{g_{\pi\mathrm{N}\Delta}}{\sqrt{6}M_{\mathrm{N}}}\Bigg[\frac{1}{\sqrt{2}}\bar{\Delta}^+_\omega(x)\Theta^{\omega\varphi}n(x)\partial_\varphi\pi^+(x) \\ &\quad - \frac{1}{\sqrt{2}}\bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^-(x) \\ &\quad - \bar{\Delta}^+_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^0(x) \\ &\quad - \bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^0(x) + \ldots\Bigg],\end{aligned} \tag{1.7}$$

where $M_{\mathrm{N}} = M_{\mathrm{n}} = M_{\mathrm{p}} = 940\,\mathrm{MeV}$ is the nucleon mass. The nucleon field $N(x)$ is the isotopical doublet with the components $N(x) = (p(x), n(x))$, and $\pi^a(x)$ is the pion field with the components $\pi^1(x) = (\pi^-(x) + \pi^+(x))/\sqrt{2}$, $\pi^2(x) = (\pi^-(x) - \pi^+(x))/i\sqrt{2}$ and $\pi^3(x) = \pi^0(x)$. The tensor $\Theta^{\omega\varphi}$ is given in ref. [15]: $\Theta^{\omega\varphi} = g^{\omega\varphi} - (Z + 1/2)\gamma^\omega\gamma^\varphi$, where the parameter Z is arbitrary. There is no consensus on the exact value of Z. From the theoretical point of view $Z = 1/2$ is preferred [15]. Phenomenological studies give only the bound $|Z| \le 1/2$ [18]. The value of the coupling constant $g_{\pi N\Delta}$ relative to the coupling constant $g_{\pi NN}$ is $g_{\pi N\Delta} = 2\, g_{\pi NN}$ [19].

Assuming that the transition $\Delta \to \mathrm{N} + \gamma$ is primarily a magnetic one the effective Lagrangian describing the $\Delta \to \mathrm{N} + \gamma$ decays can be determined as [19–21]

$$\begin{aligned}\mathcal{L}_{\gamma N\Delta}(x) &= ie\frac{g_{\gamma N\Delta}}{2M_{\rm N}}\bar{N}(x)\gamma_\alpha\gamma^5\Delta^3_\beta(x)F^{\beta\alpha}(x) + \text{h.c.}\\ &= -\frac{ie}{\sqrt{6}}\frac{g_{\gamma N\Delta}}{M_{\rm N}}[\bar{p}(x)\gamma_\alpha\gamma^5\Delta^+_\beta(x)\\ &\quad +\bar{n}(x)\gamma_\alpha\gamma^5\Delta^0_\beta(x)]F^{\beta\alpha}(x) + \text{h.c.},\end{aligned} \tag{1.8}$$

where $F^{\alpha\beta}(x) = \partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x)$ is the electromagnetic strength field tensor and $A^\alpha(x)$ is a vector potential of the electromagnetic field. The value of the coupling constant $g_{\gamma N\Delta}$ relative to the coupling constant $g_{\pi NN}$ is $g_{\gamma N\Delta} = 0.14\, g_{\pi NN}$ caused by the $SU(6)$ symmetry of strong low-energy interactions [19].

The NNJL model realizes the Lagrange approach to the description of low-energy nuclear forces [1]. For the evaluation of the effective Lagrangian of the transition n + p → D + γ it is necessary, first, to determine the effective Lagrangian of the strong low-energy transition n + p → n + p, or more generally N + N → N + N, where N = (p, n) is a nucleon field. Since the NNJL model describes low-energy interactions of the deuteron in terms of one-nucleon loop exchanges the effective Lagrangian of the transition N + N → N + N plays an important role. Due to the transition n + p → n + p the np pair on-mass shell in the initial state transfers itself into the np pair off-mass shell couples to the deuteron and the photon through one-nucleon loop exchanges. Then, the one-nucleon loop diagrams are calculated at leading order in the $1/M_{\rm N}$ expansion that corresponds to the large $N_{\rm C}$ expansion due to the proportionality $M_{\rm N} \sim N_{\rm C}$ valid in the multi-colour QCD with $SU(N_{\rm C})$ gauge group at $N_{\rm C} \to \infty$ [22].

Such a procedure of the evaluation of effective Lagrangians in the NNJL model resembles that has being used in the ENJL model [13,23–25] for the derivation of effective chiral Lagrangians up to the formal replacement $\mathrm{q}(\bar{\mathrm{q}}) \to \mathrm{N}(\bar{\mathrm{N}})$, where $\mathrm{q}(\bar{\mathrm{q}})$ is a quark (antiquark) field. In the ENJL model the dominance of the leading order contributions in the $1/M_{\rm q}$ expansion, where $M_{\rm q}$ is a constituent quark mass, has been explained by a dynamics of quark confinement, whereas in the NNJL model the dominance of the leading contributions in the $1/M_{\rm N}$ expansion is justified by the large $N_{\rm C}$ approach to non-perturbative QCD.

Since relative momenta of the np pair in the reaction of the M1-capture are smaller compared with the mass of the pion $M_\pi = 135\,\mathrm{MeV}$, for the derivation of the effective Lagrangian of the strong low-energy transition n + p → n + p we can follow the ideology of the EFT [7–12] and integrate out pion degrees of freedom as well as other heavier degrees of freedom. The result of the integration can be represented in the form of two contributions, where the first is the explicit one-pion exchange, whereas the second is a phenomenological one describing a collective contribution of both many-pion exchanges and heavier meson degrees of freedom such as $\sigma(660)$, $\rho(770)$, $\omega(782)$ and so on.

The effective Lagrangian $\mathcal{L}^{\rm np\to np}_{\text{one-pion}}(x)$ of the strong low-energy transition n + p → n + p in the one-pion exchange approximation is defined by

$$\begin{aligned}\mathcal{L}^{\rm np\to np}_{\text{one-pion}}(x) = \frac{g^2_{\pi NN}}{M^2_\pi}\,&\{[\bar{n}(x)\gamma^5 n(x)][\bar{p}(x)\gamma^5 p(x)]\\ &-2\,[\bar{p}(x)\gamma^5 n(x)][\bar{n}(x)\gamma^5 p(x)]\},\end{aligned} \tag{1.9}$$

where $g_{\pi NN} = 13.4$ is the πNN coupling constant, $n(x)$ and $p(x)$ are the operators of the interpolating fields of the neutron and the proton. The effective Lagrangian eq. (1.9) is local, as we have neglected the squared momentum transfer $-q^2$ with respect to the squared pion mass, $-q^2 \ll M^2_\pi$. Since in the reaction n + p → D + γ the np pair couples to the deuteron and the photon in the $^1S_0$ state, we should rearrange the operators of the neutron and the proton interpolating fields in the effective interaction eq. (1.9) by such a way to introduce the products of the np operators creating the np states with definite total spins. This rearrangement can be carried out by means of the Fierz transformation that gives

$$\begin{aligned}\gamma^5\otimes\gamma^5 = \frac{1}{4}\,C\otimes C + \frac{1}{4}\,\gamma^5 C\otimes C\gamma^5 + \frac{1}{4}\,\gamma^\mu C\otimes C\gamma_\mu\\ +\frac{1}{4}\,\gamma^\mu\gamma^5 C\otimes C\gamma_\mu\gamma^5 + \frac{1}{8}\,\sigma^{\mu\nu}C\otimes C\sigma_{\mu\nu},\end{aligned} \tag{1.10}$$

where $\sigma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$. By virtue of eq. (1.10) we recast the effective Lagrangian eq. (1.9) into the form

$$\begin{aligned}\mathcal{L}^{\rm np\to np}_{\text{one-pion}}(x) = \frac{g^2_{\pi NN}}{4M^2_\pi}\,&\{[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]\\ &+[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)]\\ &+[\bar{p}(x)n^c(x)][\bar{n^c}(x)p(x)]\\ &+3\,[\bar{p}(x)\gamma^\mu n^c(x)][\bar{n^c}(x)\gamma_\mu p(x)]\\ &+\frac{3}{2}\,[\bar{p}(x)\sigma^{\mu\nu}n^c(x)][\bar{n^c}(x)\sigma_{\mu\nu}p(x)]\}.\end{aligned} \tag{1.11}$$

Here $\bar{n^c}(x) = n^T(x)C$ and $n^c(x) = C\bar{n}^T(x)$, where C is a charge conjugation matrix and T is a transposition. The first two terms in the effective Lagrangian eq. (1.11) describe the strong low-energy $^1S_0 \to {}^1S_0$ transition of the np pair in the $^1S_0$ state. Thereby, the effective Lagrangian providing the $^1S_0 \to {}^1S_0$ transition of the np pair and caused by the one-pion exchange we would use in the form

$$\begin{aligned}\mathcal{L}^{\rm np\to np}_{\text{one-pion}}(x) = \frac{g^2_{\pi NN}}{4M^2_\pi}\,&\{[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]\\ &+[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)]\}.\end{aligned} \tag{1.12}$$

The phenomenological part of the effective Lagrangian responsible for the strong low-energy $^1S_0 \to {}^1S_0$ transition of the np pair in the $^1S_0$ state we would choose by following the EFT ideology [7–12] as well and write

$$\begin{aligned}\mathcal{L}^{\rm np\to np}_{\rm ph.}(x) = -\frac{2\pi a_{\rm np}}{M_{\rm N}}\,&\{[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]\\ &+[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)]\},\end{aligned} \tag{1.13}$$

where $a_{\rm np} = (-23.75 \pm 0.01)\,{\rm fm}$ is the S wave scattering length of the elastic np scattering in the ${}^1{\rm S}_0$ state [17].

The appearance of the S wave scattering length for the definition of the phenomenological coupling constant of the strong low-energy transition n + p → n + p, or differently low-energy elastic np scattering is rather natural in the EFT. Indeed, in the EFT pions are treated perturbatively and at leading order in the pion-exchange approximation for the description of the S wave scattering length of low-energy elastic np scattering in the ${}^1S_0$ state Weinberg has introduced a phenomenological four-nucleon interaction with a coupling constant [11]

$$C_0 = -\frac{4\pi a_{\rm np}}{M_{\rm N}}. \qquad (1.14)$$

As has been stated in the EFT this constant is a collective contribution coming from the integration over all meson degrees of freedom heavier than the pionic ones. Since in our approach we separate the pionic degrees of freedom from that with masses heavier than the pion as well, the integration over these heavy meson degrees of freedom should have the same form as it is postulated in the EFT. We have only halved the coupling constant C_0 defined by eq. (1.14) in order to distribute symmetrically phenomenological contributions between couplings $[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]$ and $[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)]$.

The total effective Lagrangian describing the strong low-energy transition n + p → n + p of the np pair in the ${}^1S_0$ state is then determined by

$$\begin{aligned}\mathcal{L}^{\rm np\to np}_{\rm eff}(x) &= \mathcal{L}^{\rm np\to np}_{\rm one\text{-}pion}(x) + \mathcal{L}^{\rm np\to np}_{\rm ph.}(x) = \\ &= C_{\rm NN}\,\{[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)] \\ &\quad + [\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)]\}, \quad (1.15)\end{aligned}$$

where the effective coupling constant $C_{\rm NN}$ of the strong low-energy transition n + p → n + p is equal to

$$C_{\rm NN} = \frac{g^2_{\pi \rm NN}}{4M^2_\pi} - \frac{2\pi a_{\rm np}}{M_{\rm N}} = 3.27\times 10^{-3}\,{\rm MeV}^{-2}. \qquad (1.16)$$

Note that the contribution of the phenomenological part to the effective coupling constant $C_{\rm NN}$ makes up less than 33%.

Since nuclear forces are isotopically invariant [26], the effective Lagrangian of the strong low-energy N + N → N + N transition of the NN pair in the ${}^1S_0$ state can be defined as follows:

$$\begin{aligned}\mathcal{L}^{\rm NN\to NN}_{\rm eff}(x) &= C_{\rm NN}\,\{[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)] \\ &\quad + [\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 p(x)] \\ &\quad + \frac{1}{2}[\bar{n}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 n(x)] \\ &\quad + [\bar{n}(x)\gamma^\mu\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\mu\gamma^5 n(x)] \\ &\quad + \frac{1}{2}[\bar{p}(x)\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^5 p(x)] \\ &\quad + [\bar{p}(x)\gamma^\mu\gamma^5 p^c(x)][\bar{p^c}(x)\gamma_\mu\gamma^5 p(x)]\}. \quad (1.17)\end{aligned}$$

We would like to emphasize the effective Lagrangian eq. (1.16) describing strong low-energy N + N → N + N transitions is obtained in complete agreement with the EFT ideology.

In the low-energy limit the effective local four-nucleon interaction eq. (1.17) vanishes due to the reduction

$$\begin{aligned}&[\bar{N}(x)\gamma_\mu\gamma^5 N^c(x)][\bar{N}^c(x)\gamma^\mu\gamma^5 N(x)] \to \\ &\quad -[\bar{N}(x)\gamma^5 N^c(x)][\bar{N}^c(x)\gamma^5 N(x)], \qquad (1.18)\end{aligned}$$

where $N(x)$ is an operator of the neutron or the proton interpolating field. Such a vanishing of the one-pion exchange contribution to the NN potential is well-known in the EFT approach [11,12] and the PMA [26]. In power counting [11,12] the interaction induced by the one-pion exchange is of order $O(p^2)$, where p is a relative momentum of the NN system. The former is due to the Dirac matrix γ^5 which leads to the interaction between small components of Dirac bispinors of nucleon wave functions.

In the one-nucleon loop exchange approach the contributions of the interactions $[\bar{N}(x)\gamma_\mu\gamma^5 N^c(x)]\cdot[\bar{N}^c(x)\gamma^\mu\gamma^5 N(x)]$ and $[\bar{N}(x)\gamma^5 N^c(x)][\bar{N}^c(x)\gamma^5 N(x)]$ to the amplitudes of nuclear reactions are different and do not cancel each other in the low-energy limit due to the dominance of nucleon-loop anomalies [1]. This provides the interaction between large components of Dirac bispinors of nucleon wave functions that distinguishes the NNJL model from the EFT.

1.2 Low-energy weak nuclear reactions with the deuteron

The weak nuclear reaction p + p → D + e^+ + $\nu_{\rm e}$, the solar proton burning, plays an important role in Astrophysics [3,27]. It gives start for the p-p chain of nucleosynthesis in the Sun and main-sequence stars [3,27]. In the Standard Solar Model (SSM) [28] the total (or bolometric) luminosity of the Sun $L_\odot = (3.846 \pm 0.008)\times 10^{26}\,{\rm W}$ is normalized to the astrophysical factor $S_{\rm pp}(0)$ for the solar proton burning. The recommended value $S_{\rm pp}(0) = 4.00\times 10^{-25}\,{\rm MeVb}$ [29] has been found by averaging over the results obtained in the Potential model approach (PMA) [30,31] and the Effective Field Theory (EFT) approach [32,33]. As has been shown recently in ref. [34] *the inverse and forward helioseismic approach* confirm the recommended value of $S_{\rm pp}(0)$ within experimental errors on the helioseismic data and solar neutrino fluxes.

In this paper we apply the NNJL model to the description of low-energy nuclear forces for weak nuclear reactions with the deuteron of astrophysical interest: 1) the solar proton burning p + p → D + e^+ + $\nu_{\rm e}$, 2) the pep-process p + e^- + p → D + $\nu_{\rm e}$ and 3) the reactions of neutrino and antineutrino disintegration of the deuteron caused by charged $\nu_{\rm e}$ + D → e^- + p + p, $\bar{\nu}_{\rm e}$ + D → e^+ + n + n and neutral $\nu_{\rm e}(\bar{\nu}_{\rm e})$ + D → $\nu_{\rm e}(\bar{\nu}_{\rm e})$ + n + p weak currents. The reactions $\nu_{\rm e}$ + D → e^- + p + p and $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p caused by charged and neutral weak currents, respectively, and induced by solar neutrinos are planned to be measured for solar neutrino experiments at

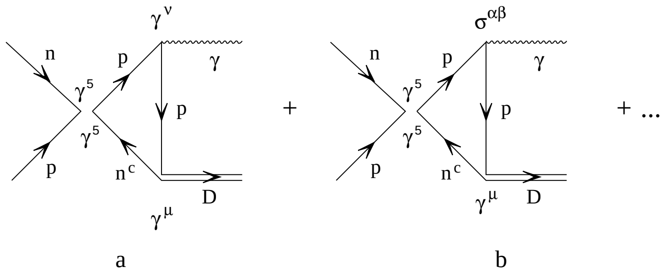

Fig. 1. One-nucleon loop diagrams of the contribution of the effective coupling $[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]$ to the effective Lagrangian of the M1 transition n + p → D + γ.

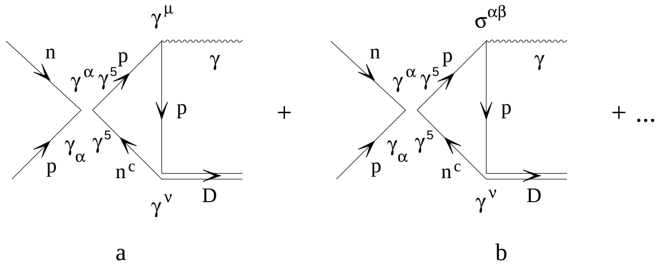

Fig. 2. One-nucleon loop diagrams of the contribution of the effective coupling $[\bar{p}(x)\gamma^\alpha\gamma^5 n^c(x)][\bar{n^c}(x)\gamma_\alpha\gamma^5 p(x)]$ to the effective Lagrangian of the M1 transition n + p → D + γ.

Sudbury Neutrino Observatory (SNO) [35]. In turn, the cross-sections for the reactions $\bar{\nu}_{\rm e}$ + D → e$^+$ + n + n and $\bar{\nu}_{\rm e}$ + D → $\bar{\nu}_{\rm e}$ + n + p caused by charged and neutral weak currents, respectively, and induced by reactor antineutrinos have been recently measured by the Reines's experimental group [36]. In the sense of charge independence of weak interaction strength the observation of the reaction $\bar{\nu}_{\rm e}$ + D → e$^+$ + n + n is equivalent to the observation of the reaction of the solar proton burning p + p → D + e$^+$ + $\nu_{\rm e}$ in the terrestrial laboratories.

The paper is organized as follows. In sect. 2 we evaluate the amplitude of the M1-capture n + p → D + γ in the tree-meson approximation. The contribution of low-energy elastic np scattering to the amplitude of the process n + p → D + γ is obtained in agreement with low-energy nuclear phenomenology. In sect. 3 we evaluate contributions of chiral one-meson loop corrections to the amplitude of the M1-capture in chiral perturbation theory at the quark level (CHPT)$_{\rm q}$ developed within the ENJL model with a linear realization of chiral $U(3)\times U(3)$ symmetry. In sect. 4 we include the contribution of the $\Delta(1232)$ resonance and analyse the total cross-section for the neutron-proton radiative capture for thermal neutrons and compare it with experimental data. In sect. 5 we treat the reaction of the photomagnetic disintegration of the deuteron γ + D → n + p related to the neutron-proton radiative capture n + p → D + γ via time-reversal invariance and analyse the energy dependence of the cross-section at energies far from threshold. In sect. 6 we evaluate the amplitude of the solar proton burning. We show that the contribution of low-energy elastic pp scattering in the $^1S_0$ state with the Coulomb repulsion is described in agreement with low-energy nuclear phenomenology in terms of the S wave scattering length and the effective range. In sect. 7 we evaluate the astrophysical factor for the solar proton burning and obtain the value $S_{\rm pp}(0) = 4.08\times 10^{-25}\,{\rm MeV\,b}$ agreeing well with the recommended one $S_{\rm pp}(0) = 4.00\times 10^{-25}\,{\rm MeV\,b}$. In sect. 8 we evaluate the cross-section for the neutrino disintegration of the deuteron $\nu_{\rm e}$ + D → e$^-$ + p + p with respect to $S_{\rm pp}(0)$. In sect. 9 we adduce the evaluation of the astrophysical factor $S_{\rm pep}(0)$ for the pep-process relative to $S_{\rm pp}(0)$. In sects. 10 and 11 we evaluate the cross-sections for the antineutrino disintegration of the deuteron $\bar{\nu}_{\rm e}$ + D → e$^+$ + n + n and $\bar{\nu}_{\rm e}$ + D → $\bar{\nu}_{\rm e}$ + n + p and average them over the antineutrino energy spectrum. The average values of the cross-sections agree well with experimental data [36]. The cross-sections for the weak nuclear reactions of astrophysical interest are calculated at zero contribution of the nucleon tensor current [1]. This makes the description of low-energy nuclear forces within the NNJL model compatible with the predictions of the SSM [27,28]. In more detail this is discussed the conclusion and appendix. In the conclusion we discuss the obtained results. In the appendix we evaluate the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$ of the low-energy weak transition p + p → D + e$^+$ + $\nu_{\rm e}$. The contribution of the nucleon tensor current [1] to the effective Lagrangians of low-energy weak nuclear transitions of astrophysical interest like p + p → D + e$^+$ + $\nu_{\rm e}$ and so on is evaluated and discussed.

2 The M1-capture in the tree-meson approximation

Since the NNJL model realizes a Lagrange approach to the description of low-energy nuclear forces [1], the first thing that we have to do is to evaluate the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm np\to D\gamma}(x)$ of the transition n + p → D + γ. In the tree-meson approximation the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm np\to D\gamma}(x)$ is defined by one-nucleon loop diagrams depicted in figs. 1 and 2. The evaluation of these diagrams at leading order in the large $N_{\rm C}$ expansion yields

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm np\to D\gamma}(x) &= (\mu_{\rm p}-\mu_{\rm n})\,\frac{e}{2M_{\rm N}}\,\frac{g_{\rm V}}{4\pi^2}\\ &\quad\times C_{\rm NN}\,D^{\dagger}_{\mu\nu}(x)^{*}F^{\mu\nu}(x)[\bar{p^c}(x)\gamma^5 n(x)]\\ &\quad+i\,(\mu_{\rm p}-\mu_{\rm n})\,\frac{e}{2M_{\rm N}}\,\frac{g_{\rm V}}{4\pi^2}\,C_{\rm NN}\,M_{\rm N}\\ &\quad\times D^{\dagger}_{\mu}(x)^{*}F^{\mu\nu}(x)[\bar{p^c}(x)\gamma_\nu\gamma^5 n(x)],\end{aligned}\qquad(2.1)$$

where ${}^*F^{\mu\nu}(x) = \frac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}(x)$, $\mu_{\rm p} = 2.793$ and $\mu_{\rm n} = -\,1.913$ are the magnetic dipole moments of the proton and the neutron measured in nuclear magnetons, $D_\mu(x)$ is the operator of the interpolating field of the deuteron and $D_{\mu\nu}(x) = \partial_\mu D_\nu(x) - \partial_\nu D_\mu(x)$, $g_{\rm V}$ is a phenomenological coupling constant of the NNJL model related to the electric quadrupole moment of the deuteron $Q_{\rm D} = 0.286\,{\rm fm}^2$ [1]: $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$.

The matrix element of the transition $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$ we define in the usual way

$$\int \mathrm{d}^4x \langle D(k_{\mathrm{D}})\gamma(k)|\mathcal{L}_{\mathrm{eff}}^{\mathrm{np}\to\mathrm{D}\gamma}(x)|n(p_1)p(p_2)\rangle = (2\pi)^4\delta^{(4)}(k_{\mathrm{D}}+k-p_1-p_2)\frac{\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)}{\sqrt{2E_1V\,2E_2V\,2E_{\mathrm{D}}V\,2\omega V}}, \quad (2.2)$$

where $E_i\,(i=1,2,\mathrm{D})$ and ω are the energies of the neutron, the proton, the deuteron and the photon, V is the normalization volume.

The wave functions of the initial $|n(p_1)p(p_2)\rangle$ and final $\langle D(k_{\mathrm{D}})\gamma(k)|$ state we take in the usual form

$$|n(p_1)p(p_2)\rangle = a^{\dagger}_{\mathrm{n}}(\mathbf{p}_1,\sigma_1)\,a^{\dagger}_{\mathrm{p}}(\mathbf{p}_2,\sigma_2)|0\rangle,$$
$$\langle D(k_{\mathrm{D}})\gamma(k)| = \langle 0|a_{\mathrm{D}}(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,a(\mathbf{k},\lambda), \quad (2.3)$$

where $a^{\dagger}_{\mathrm{n}}(\mathbf{p}_1,\sigma_1)$ and $a^{\dagger}_{\mathrm{p}}(\mathbf{p}_2,\sigma_2)$ are the operators of creation of the neutron and the proton, and $a_{\mathrm{D}}(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})$ and $a(\mathbf{k},\lambda)$ are the operators of annihilation of the deuteron and the photon.

The matrix element of the transition $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$ reads

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma) = = (\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\frac{e}{2M_{\mathrm{N}}}\frac{g_{\mathrm{V}}}{4\pi^2}\,C_{\mathrm{NN}}\,\varepsilon^{\alpha\beta\mu\nu}k_{\alpha}\,e^*_{\beta}(k,\lambda)\,e^*_{\mu}(k_{\mathrm{D}},\lambda_{\mathrm{D}}) \times[\bar{u^c}(p_2)(2\,k_{\mathrm{D}\nu}-M_{\mathrm{N}}\gamma_{\nu})\gamma^5u(p_1)], \quad (2.4)$$

where $e^*_{\beta}(k,\lambda)$ and $e^*_{\mu}(k_{\mathrm{D}},\lambda_{\mathrm{D}})$ are the 4-vectors of the polarization of the photon and the deuteron, then $\bar{u^c}(p_2)$ and $u(p_1)$ are the bispinorial wave functions of the proton and the neutron, respectively, normalized by $\bar{u^c}(p_2)u^c(p_2) = -\,2\,M_{\mathrm{N}}$ and $\bar{u}(p_1)u(p_1) = 2\,M_{\mathrm{N}}$.

In the low-energy limit when

$$[\bar{u^c}(p_2)\gamma_{\nu}\gamma^5u(p_1)] \to -g_{\nu 0}\,[\bar{u^c}(p_2)\gamma^5u(p_1)] \quad (2.5)$$

and $k_{\mathrm{D}\nu} \to g_{\nu 0}\,2M_{\mathrm{N}}$ the matrix element eq. (2.4) acquires the form

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2} \times C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})[\bar{u^c}(p_2)\gamma^5u(p_1)]. \quad (2.6)$$

The evaluation of the matrix element eq. (2.6), the effective vertex of the $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$ transition, we have carried out with the wave functions of the neutron and the proton in the form of the plane waves. However, a physical $^1S_0$ state of the np pair is defined by low-energy nuclear forces. For the description of the contribution of low-energy nuclear forces to the physical $^1S_0$ state of the np pair coupled to the deuteron and the photon we suggest to sum an infinite series of one-nucleon bubbles with vertices defined by the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{\mathrm{np}\to\mathrm{np}}(x)$ eq. (1.15). The result of the summation can be represented in the following form

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2} C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})[\bar{u^c}(p_2)\gamma^5u(p_1)] \times\frac{1}{1+\dfrac{C_{\mathrm{NN}}}{16\pi^2}\displaystyle\int\frac{\mathrm{d}^4q}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{Q}}\Big\}}, \quad (2.7)$$

where $P = p_1+p_2 = (2\sqrt{p^2+M^2_{\mathrm{N}}},\mathbf{0}\,)$ is the 4-momentum of the np pair in the center-of-mass frame. Then, $Q = a\,P + b\,K = a\,(p_1+p_2) + b\,(p_1-p_2)$ is an arbitrary shift of virtual momentum with arbitrary parameters a and b, and in the center-of-mass frame $K = p_1 - p_2 = (0, 2\,\mathbf{p}\,)$. The explicit dependence of the momentum integral on Q can be evaluated by means of the Gertsein-Jackiw procedure [37] (see also ref. [1]). It is given by

$$\int\frac{\mathrm{d}^4q}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{Q}}\Big\} = \int\frac{\mathrm{d}^4q}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{P}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}}\Big\} -2\,a\,(a+1)\,P^2 - 2\,b^2\,K^2. \quad (2.8)$$

For the evaluation of the momentum integral over q we would keep only the leading order contributions in the $1/M_{\mathrm{N}}$ expansion caused by the large N_{C} expansion [1]. This yields

$$\int\frac{\mathrm{d}^4q}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{q}-\hat{Q}}\Big\} = -8a(a+1)M^2_{\mathrm{N}} + 8\,(b^2-a\,(a+1))\,p^2 - i\,8\pi\,M_{\mathrm{N}}p. \quad (2.9)$$

The expression eq. (2.7) we reduce to the form

$$\mathcal{M}\;(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}} \times(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,[\bar{u^c}(p_2)\gamma^5u(p_1)] \times\frac{\mathcal{Z}}{1-\frac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2 + ia_{\mathrm{np}}p}. \quad (2.10)$$

Here we have denoted

$$a_{\mathrm{np}} = -\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,\mathcal{Z}\quad,$$
$$r_{\mathrm{np}} = (b^2-a\,(a+1))\,\frac{2}{\pi}\,\frac{1}{M_{\mathrm{N}}},$$
$$\frac{1}{\mathcal{Z}} = 1 - \frac{a(a+1)}{2\pi^2}\,C_{\mathrm{NN}}\,M^2_{\mathrm{N}}, \quad (2.11)$$

where $r_{\mathrm{np}} = 2.75 \pm 0.05\,\mathrm{fm}$ is the effective range of low-energy elastic np scattering [17].

Renormalizing the wave functions of nucleons $\sqrt{\mathcal{Z}}\,u(p_1) \to u(p_1)$ and $\sqrt{\mathcal{Z}}\,u(p_2) \to u(p_2)$, we arrive at the expression

$$\mathcal{M}\ (\mathrm{n}+\mathrm{p}\to \mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}} \times(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,[\bar{u^c}(p_2)\gamma^5 u(p_1)] \times\frac{1}{1-\frac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2+i\,a_{\mathrm{np}}p}, \quad (2.12)$$

where the factor $1/(1-\frac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2+i\,a_{\mathrm{np}}p)$ describes the contribution of low-energy nuclear forces to the physical ${}^1S_0$ state of the np pair coupled to the deuteron and the photon. It has the form of the amplitude of low-energy elastic np scattering in the ${}^1S_0$ state in complete agreement with low-energy nuclear phenomenology [26]. By using the relation expressing the phase shift $\delta_{\mathrm{np}}(p)$ of low-energy elastic np scattering in terms of the S wave scattering length a_{np} and the effective range r_{np} [26]

$$\mathrm{ctg}\,\delta_{\mathrm{np}}(p) = -\frac{1}{a_{\mathrm{np}}}+\frac{1}{2}\,r_{\mathrm{np}}\,p^2, \quad (2.13)$$

we can recast the factor $1/(1-\frac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2+i\,a_{\mathrm{np}}p)$ into the form

$$\frac{1}{1-\frac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2+i\,a_{\mathrm{np}}p} = e^{i\delta_{\mathrm{np}}(p)}\frac{\sin\delta_{\mathrm{np}}(p)}{-a_{\mathrm{np}}\,p}. \quad (2.14)$$

In terms of the phase shift $\delta_{\mathrm{np}}(p)$ the expression eq. (2.12) reads

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to \mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}}) \times[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,e^{i\delta_{\mathrm{np}}(p)}\frac{\sin\delta_{\mathrm{np}}(p)}{-a_{\mathrm{np}}\,p}. \quad (2.15)$$

In the NNJL model low-energy nuclear forces between the neutron and the proton in the physical deuteron state are described by the one-nucleon loop exchanges in terms of the phenomenological coupling constant g_{V} which is defined by the electric quadrupole moment of the deuteron Q_{D}, $g^2_{\mathrm{V}} = 2\pi^2 Q_{\mathrm{D}}M^2_{\mathrm{N}}$ [1]. The electric quadrupole moment of the deuteron is caused by nuclear tensor forces [26]. Therefore, the relation $g^2_{\mathrm{V}} = 2\pi^2 Q_{\mathrm{D}}M^2_{\mathrm{N}}$ confirms at the quantum field-theoretic level the fact pointed out by Blatt and Weisskopf that *the existence of a bound triplet state of the neutron-proton system would be entirely due to the tensor force* [38]. Thus, in NNJL model through the phenomenological coupling constant g_{V} tensor forces govern the existence of the deuteron as a bound neutron-proton triplet spin state and strength of low-energy interactions of the deuteron with nucleons and other particles in terms of the one-nucleon loop exchanges. The evaluation of one-nucleon loop diagrams at leading order in the $1/M_{\mathrm{N}}$ expansion, or in the large N_{C} expansion, reduces a momentum dependence of nucleon diagrams to the trivial form accounting for only the Lorentz covariant properties of the interaction. In this approach the deuteron looks like a point-like particle. Such a representation is enough for the evaluation of effective Lagrangians of different low-energy nuclear transitions with the deuteron in an initial or a final state describing effective vertices of low-energy nuclear transitions defined at their thresholds. However, for the evaluation of amplitudes of low-energy nuclear reactions for energies far from threshold one needs to take into account a spatial smearing of the physical deuteron caused by a finite radius $r_{\mathrm{D}} = 1/\sqrt{\varepsilon_{\mathrm{D}}M_{\mathrm{N}}} = 4.319\,\mathrm{MeV}$ [17] determined by the binding energy of the deuteron $\varepsilon_{\mathrm{D}} = 2.225\,\mathrm{MeV}$. The spatial smearing of the physical deuteron we introduce phenomenologically in the form

$$F_{\mathrm{D}}(p) = \frac{1}{1+r^2_{\mathrm{D}}p^2}, \quad (2.16)$$

that is nothing more than the momentum representation of the approximate ${}^3S_1$ wave state of the deuteron [26].

Substituting eq. (2.16) in eq. (2.15), we obtain the amplitude of the M1-capture calculated in the NNJL model:

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to \mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}}) \times[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,e^{i\delta_{\mathrm{np}}(p)}\frac{\sin\delta_{\mathrm{np}}(p)}{-a_{\mathrm{np}}\,p}\,\frac{1}{1+r^2_{\mathrm{D}}p^2}. \quad (2.17)$$

For thermal neutrons the kinetic energy of the relative movement of the np pair is of order $T_{\mathrm{np}}\sim 10^{-8}\,\mathrm{MeV}$. This yields the relative momentum of the np pair to be smaller compared with the binding energy of the deuteron, $p\sim 3\times 10^{-3}\,\mathrm{MeV}\ll\varepsilon_{\mathrm{D}} = 2.225\,\mathrm{MeV}$. Therefore, for thermal neutrons without loss of generality we can calculate the amplitude of the M1-capture setting $p=0$:

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to \mathrm{D}+\gamma) = e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}}) \times[\bar{u^c}(p_2)\gamma^5 u(p_1)]. \quad (2.18)$$

Thus, we have obtained that the amplitude of the neutron-proton radiative capture $\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma$ for thermal neutrons coincides with the matrix element of the effective Lagrangian $\mathcal{L}^{\mathrm{np}\to\mathrm{D}\gamma}_{\mathrm{eff}}(x)$ given by eq. (2.6).

The cross-section for the M1-capture for thermal neutrons calculated in the tree-meson approximation is then defined by

$$\sigma(\mathrm{np}\to\mathrm{D}\gamma)(T_{\mathrm{n}}) = \frac{1}{v_{\mathrm{n}}}\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})^2\,\frac{25}{64}\,\frac{\alpha}{\pi^2}\,Q_{\mathrm{D}}\,C^2_{\mathrm{NN}}\,M_{\mathrm{N}}\,\varepsilon^3_{\mathrm{D}}\left(1+\frac{1}{2}\,\frac{T_{\mathrm{n}}}{\varepsilon_{\mathrm{D}}}\right)^3 = 276\,\mathrm{mb}. \quad (2.19)$$

The theoretical value $\sigma(\mathrm{np}\to\mathrm{D}\gamma)(T_{\mathrm{n}}) = 276$ mb is about 17% smaller compared with the experimental one $\sigma(\mathrm{np}\to\mathrm{D}\gamma)_{\exp}(T_{\mathrm{n}}) = (334.2\pm 0.5)\,\mathrm{mb}$. Thus, in the tree-meson approximation the NNJL model predicts the cross-section for the M1-capture for thermal neutrons somewhat worse than the EFT, $\sigma(\mathrm{np}\to \mathrm{D}\gamma)(T_\mathrm{n}) = 297.2\,\mathrm{mb}$ [8]. In order to improve the agreement with the experimental data we have to include chiral one-meson loop corrections [7–9] and the contribution of the $\Delta(1232)$ resonance [6,7].

3 Chiral one-meson loop corrections to the amplitude of the M1-capture

For the evaluation of chiral meson-loop corrections in the NNJL we use $(\mathrm{CHPT})_\mathrm{q}$ developed in refs. [13] within the ENJL model with a linear realization of chiral $U(3)\times U(3)$ symmetry. Below we consider the contributions of chiral one-meson loop corrections induced by the virtual meson transitions $\pi\to a_1\gamma$, $a_1\to\pi\gamma$, $\pi\to(\omega,\rho)\gamma$, $(\omega,\rho)\to\pi\gamma$, $\sigma\to(\omega,\rho)\gamma$ and $(\omega,\rho)\to\sigma\gamma$, where σ is a scalar partner of pions under chiral $SU(2)\times SU(2)$ transformations in $(\mathrm{CHPT})_\mathrm{q}$ with a linear realization of chiral $U(3)\times U(3)$ symmetry [13].

The effective Lagrangians $\delta\mathcal{L}^{\mathrm{pp}\gamma}_{\mathrm{eff}}(x)$ and $\delta\mathcal{L}^{\mathrm{nn}\gamma}_{\mathrm{eff}}(x)$, caused by the virtual meson transitions $\pi\to a_1\gamma$, $a_1\to\pi\gamma$, $\pi\to(\omega,\rho)\gamma$, $(\omega,\rho)\to\pi\gamma$, $\sigma\to(\omega,\rho)\gamma$ and $(\omega,\rho)\to\sigma\gamma$, we evaluate at leading order in the large N_C expansion [1]. The results of the evaluation contain divergent contributions. In order to remove these divergences we apply the renormalization procedure developed in $(\mathrm{CHPT})_\mathrm{q}$ for the evaluation of chiral meson-loop corrections (see Ivanov in refs. [13]). Since the renormalized expressions should vanish in the chiral limit $M_\pi\to 0$ [13], only the virtual-meson transitions with intermediate π-meson give non-trivial contributions. The contributions of the virtual meson transitions with the intermediate σ-meson are found finite in the chiral limit and subtracted according to the renormalization procedure [13]. Such a cancellation of the σ-meson contributions in the one-meson loop approximation agrees with Chiral perturbation theory using a non-linear realization of chiral symmetry, where σ-meson-like exchanges can appear only in two-meson loop approximation. Then, the sum of the contributions of the virtual-meson transitions $\pi^-\to\rho^-\gamma$, $\pi^0\to\rho^0\gamma$ and $\pi^0\to\omega\gamma$ to the effective coupling $\mathrm{nn}\gamma$ is equal to zero. As a result the effective Lagrangians $\delta\mathcal{L}^{\mathrm{pp}\gamma}_{\mathrm{eff}}(x)$ and $\delta\mathcal{L}^{\mathrm{nn}\gamma}_{\mathrm{eff}}(x)$ are given by

$$\begin{aligned}\delta\mathcal{L}^{\mathrm{pp}\gamma}_{\mathrm{eff}}(x) &= \frac{ie}{4M_\mathrm{N}}\Bigg[g_\mathrm{A} g_{\pi\mathrm{NN}}\frac{\alpha_\rho}{16\pi^3}\frac{M_\mathrm{N}}{F_\pi}M^2_\pi\, J_{\pi\mathrm{a_1N}}\\ &\quad + g_{\pi\mathrm{NN}}\,\frac{N_\mathrm{C}\alpha_\rho}{16\pi^3}\frac{M_\mathrm{N}}{F_\pi}\,M^2_\pi\, J_{\pi\mathrm{VN}}\Bigg]\\ &\quad\times[\bar p(x)\sigma_{\mu\nu}p(x)]\,F^{\mu\nu}(x),\\ \delta\mathcal{L}^{\mathrm{nn}\gamma}_{\mathrm{eff}}(x) &= \frac{ie}{4M_\mathrm{N}}\Bigg[-\,g_\mathrm{A} g_{\pi\mathrm{NN}}\frac{\alpha_\rho}{16\pi^3}\,\frac{M_\mathrm{N}}{F_\pi}M^2_\pi\, J_{\pi\mathrm{a_1N}}\Bigg]\\ &\quad\times[\bar n(x)\sigma_{\mu\nu}n(x)]\,F^{\mu\nu}(x), \qquad (3.1)\end{aligned}$$

where $\alpha_\rho = g^2_\rho/4\pi = 2.91$ is the effective coupling constant of the $\rho\to\pi\pi$ decay, $F_\pi = 92.4\,\mathrm{MeV}$ is the leptonic coupling constant of pions, and $g_\mathrm{A} = 1.267$ [39]. Then, $J_{\pi\mathrm{a_1N}}$ and$J_{\pi\mathrm{VN}}$ are the momentum integrals determined by

$$\begin{aligned}J_{\pi\mathrm{a_1N}} &= \int\frac{\mathrm{d}^4p}{\pi^2}\,\frac{1}{(M^2_\pi+p^2)(M^2_{a_1}+p^2)(M^2_\mathrm{N}+p^2)}\\ &= 0.017\,M^{-2}_\pi,\\ J_{\pi\mathrm{VN}} &= \int\frac{\mathrm{d}^4p}{\pi^2}\,\frac{1}{(M^2_\pi+p^2)(M^2_\mathrm{V}+p^2)(M^2_\mathrm{N}+p^2)}\\ &= 0.024\,M^{-2}_\pi, \qquad (3.2)\end{aligned}$$

where p is Euclidean 4-momentum, $M_\mathrm{V} = M_\rho = M_\omega = 770\,\mathrm{MeV}$ [39] and $M_{a_1} = \sqrt{2}\,M_\rho$ [13].

At $N_\mathrm{C} = 3$ the cross-section for the M1-capture accounting for the contribution of the effective interaction eq. (3.1) amounts to

$$\begin{aligned}&\sigma(\mathrm{np}\to\mathrm{D}\gamma)(T_\mathrm{n}) =\\ &\frac{1}{v}\,(\mu_\mathrm{p}-\mu_\mathrm{n})^2\,\frac{25}{64}\,\frac{\alpha}{\pi^2}\;Q_\mathrm{D}\,C^2_\mathrm{NN}\,M_\mathrm{N}\,\varepsilon^3_\mathrm{D}\\ &\times\left[1+\frac{g^2_{\pi\mathrm{NN}}}{\mu_\mathrm{p}-\mu_\mathrm{n}}\,\frac{M^2_\pi}{8\pi^2}\,\frac{\alpha_\rho}{\pi}\left(J_{\pi\mathrm{a_1N}}+\frac{3}{2g_\mathrm{A}}\,J_{\pi\mathrm{VN}}\right)\right]^2\\ &= 287.2\,\mathrm{mb}, \qquad (3.3)\end{aligned}$$

where we have used the relation $g_{\pi\mathrm{NN}}\simeq g_\mathrm{A}\,M_\mathrm{N}/F_\pi$. The theoretical value of the cross-section for the neutron-proton radiative capture given by eq. (3.3) differs from the experimental one by about 14%. This discrepancy we should describe by taking into account the contribution of the $\Delta(1232)$ resonance.

4 The $\Delta(1232)$ resonance

For the evaluation of the contribution of the $\Delta(1232)$ resonance to the amplitude of the M1-capture in the NNJL model we have to obtain the effective Lagrangian $\mathcal{L}^{\mathrm{np}\to\Delta\mathrm{N}}_{\mathrm{eff}}(x)$ describing the strong low-energy $\mathrm{n}+\mathrm{p}\to\Delta + \mathrm{N}$ transition. For this aim we should use the procedure having been applied to the evaluation of the effective Lagrangian $\mathcal{L}^{\mathrm{np}\to\mathrm{np}}_{\mathrm{eff}}(x)$ given by eq. (1.15). In refs. [6,7] the evaluation of the contribution of the $\Delta(1232)$ resonance in terms of exchange currents has been carried out in the one-pion exchange approximation. Thereby, following refs. [6,7] we suppose to evaluate the effective Lagrangian $\mathcal{L}^{\mathrm{np}\to\Delta\mathrm{N}}_{\mathrm{eff}}(x)$ of the strong low-energy $\mathrm{n}+\mathrm{p}\to\Delta+\mathrm{N}$ transition in the one-pion exchange approximation. This gives

$$\begin{aligned}&\mathcal{L}^{\mathrm{np}\to\Delta\mathrm{N}}_{\mathrm{eff}}(x) = -\frac{i}{\sqrt{6}}\frac{g_{\pi\mathrm{N}\Delta}}{M_\mathrm{N}}\frac{g_{\pi\mathrm{NN}}}{4M^2_\pi}\\ &\times\int\mathrm{d}^4z\,\frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x)\,\{[\bar\Delta^+_\omega(z)\,\Theta^\omega{}_\varphi\,n^c(x)]\\ &\times[\bar{n^c}(z)\gamma^5p(x)+\bar{n^c}(x)\gamma^5p(z)]\\ &-[\bar\Delta^0_\omega(z)\,\Theta^\omega{}_\varphi\,p^c(x)]\,[\bar{n^c}(z)\gamma^5p(x)+\bar{n^c}(x)\gamma^5p(z)]\\ &+1\otimes\gamma^5\to-\gamma_\nu\otimes\gamma^\nu\gamma^5\}, \qquad (4.1)\end{aligned}$$

$$\int d^4x\, \mathcal{L}_{\rm eff}^{\rm np\to\Delta N\to D\gamma}(x) = -\int d^4x_1 d^4x_2 d^4x_3\, \langle {\rm T}(\mathcal{L}_{\rm eff}^{\rm np\to\Delta N}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\gamma N\Delta}(x_3))\rangle = -\frac{i}{6}\frac{eg_{\rm V}}{M_{\rm N}^2}\frac{g_{\pi N\Delta}}{g_{\pi NN}}\frac{g_{\gamma N\Delta}}{g_{\pi NN}}\frac{g^3_{\pi NN}}{4M_\pi^2}\times$$
$$\int d^4x_1 d^4x_2 d^4x_3 \int d^4z\, \frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x_1)\, {\rm T}([\bar{p^c}(x_1)\gamma^5 n(z) + \bar{p^c}(z)\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\Big\{\langle 0|{\rm T}([\bar{\Delta}^+_\omega(z)\,\Theta^\omega{}_\varphi\, n^c(x_1)][\bar{p^c}(x_2)\gamma^\mu n(x_2) - \bar{n^c}(x_2)\gamma^\mu p(x_2)][\bar{p}(x_3)\gamma_\beta\gamma^5\Delta^+_\alpha(x_3)])|0\rangle$$
$$-\langle 0|{\rm T}([\bar{\Delta}^0_\omega(z)\,\Theta^\omega{}_\varphi\, p^c(x)][\bar{p^c}(x_2)\gamma^\mu n(x_2) - \bar{n^c}(x_2)\gamma^\mu p(x_2)][\bar{n}(x_3)\gamma_\beta\gamma^5\Delta^0_\alpha(x_3)])|0\rangle + (\gamma^5\otimes 1\to -\gamma_\nu\gamma^5\otimes\gamma^\nu)\Big\} =$$
$$\frac{i}{3}\frac{eg_{\rm V}}{M_{\rm N}}\frac{g_{\pi N\Delta}}{g_{\pi NN}}\frac{g_{\gamma N\Delta}}{g_{\pi NN}}\frac{g^3_{\pi NN}}{4M_\pi^2}\int d^4x_1 d^4x_2 d^4x_3 \int d^4z\, \frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x_1)\, {\rm T}([\bar{p^c}(x_1)\gamma^5 n(z) + \bar{p^c}(z)\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\Big\{\langle 0|{\rm T}([\bar{\Delta}^+_\omega(z)\,\Theta^\omega{}_\varphi\, n^c(x_1)][\bar{n^c}(x_2)\gamma^\mu p(x_2)][\bar{p}(x_3)\gamma_\beta\gamma^5\Delta^+_\alpha(x_3)])|0\rangle$$
$$+\langle 0|{\rm T}([\bar{\Delta}^0_\omega(z)\,\Theta^\omega{}_\varphi\, p^c(x_1)][\bar{p^c}(x_2)\gamma^\mu n(x_2)][\bar{n}(x_3)\gamma_\beta\gamma^5\Delta^0_\alpha(x_3)])|0\rangle + (\gamma^5\otimes 1\to -\gamma_\nu\gamma^5\otimes\gamma^\nu)\Big\} =$$
$$= \frac{2}{3}\frac{ieg_{\rm V}}{M_{\rm N}^2}\frac{g_{\pi N\Delta}}{g_{\pi NN}}\frac{g_{\gamma N\Delta}}{g_{\pi NN}}\frac{g^3_{\pi NN}}{4M_\pi^2}\int d^4x_1 d^4x_2 d^4x_3 \int d^4z\, \frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x_1)\Big\{{\rm T}([\bar{p^c}(x_1)\gamma^5 n(z) + \bar{p^c}(z)\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\frac{1}{i}{\rm tr}\{S_{\alpha\omega}(x_3-z)\,\Theta^\omega{}_\varphi\, S^c_{\rm F}(x_1-x_2)\gamma^\mu S_{\rm F}(x_2-x_3)\gamma_\beta\gamma^5\} - {\rm T}([\bar{p^c}(x_1)\gamma_\nu\gamma^5 n(z) + \bar{p^c}(z)\gamma_\nu\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\frac{1}{i}{\rm tr}\{S_{\alpha\omega}(x_3-z)\,\Theta^\omega{}_\varphi\,\gamma^\nu S^c_{\rm F}(x_1-x_2)\gamma^\mu S_{\rm F}(x_2-x_3)\gamma_\beta\gamma^5\}\Big\}. \qquad (4.3)$$

where we have kept only the terms contributing to the transition of the np pair in the $^1S_0$ state into the ΔN state. Using then the phenomenological Lagrangian

$$\mathcal{L}_{\rm npD}(x) = -\,i\,g_{\rm V}[\bar{p^c}(x)\gamma^\mu n(x) - \bar{n^c}(x)\gamma^\mu p(x)]D^\dagger_\mu(x) \qquad (4.2)$$

the effective Lagrangian describing the contribution of the $\Delta(1232)$ resonance to the low-energy transition n + p → D + γ is defined by

see equation (4.3) *above*

Thus, the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm np\to\Delta N\to D\gamma}(x)$ is equal to

$$\int d^4x\, \mathcal{L}_{\rm eff}^{\rm np\to\Delta N\to D\gamma}(x) = \frac{2}{3}\frac{ieg_{\rm V}}{M_{\rm N}^2}\frac{g_{\pi N\Delta}}{g_{\pi NN}}\frac{g_{\gamma N\Delta}}{g_{\pi NN}}\frac{g^3_{\pi NN}}{4M_\pi^2}$$
$$\times\int d^4x_1 d^4x_2 d^4x_3 \int d^4z\, \frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x_1)$$
$$\times\Big\{{\rm T}([\bar{p^c}(x_1)\gamma^5 n(z) + \bar{p^c}(z)\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\frac{1}{i}{\rm tr}\{S_{\alpha\omega}(x_3-z)\,\Theta^\omega{}_\varphi\, S^c_{\rm F}(x_1-x_2)\gamma^\mu S_{\rm F}(x_2-x_3)\gamma_\beta\gamma^5\}$$
$$-{\rm T}([\bar{p^c}(x_1)\gamma_\nu\gamma^5 n(z)+\bar{p^c}(z)\gamma_\nu\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)F^{\alpha\beta}(x_3))$$
$$\times\frac{1}{i}{\rm tr}\{S_{\alpha\omega}(x_3-z)\,\Theta^\omega{}_\varphi\,\gamma^\nu S^c_{\rm F}(x_1-x_2)\gamma^\mu$$
$$\times S_{\rm F}(x_2-x_3)\gamma_\beta\gamma^5\}\Big\}. \qquad (4.4)$$

In the momentum representation of the baryon Green functions the effective Lagrangian eq. (4.4) reads

$$\int d^4x\, \mathcal{L}_{\rm eff}^{\rm np\to\Delta N\to D\gamma}(x) = \frac{2}{3}\frac{ieg_{\rm V}}{M_{\rm N}^2}\frac{g_{\pi N\Delta}}{g_{\pi NN}}\frac{g_{\gamma N\Delta}}{g_{\pi NN}}\frac{g^3_{\pi NN}}{4M_\pi^2}$$
$$\times\int d^4x_1 \int d^4z\, \frac{\partial}{\partial z_\varphi}\delta^{(4)}(z-x_1)$$
$$\times\int \frac{d^4x_2 d^4k_2}{(2\pi)^4}\frac{d^4x_3 d^4k_3}{(2\pi)^4}\, e^{-ik_2\cdot(x_2-x_1)}$$
$$\times e^{-ik_3\cdot(x_3-z)}\Big\{{\rm T}([\bar{p^c}(x_1)\gamma^5 n(z) + \bar{p^c}(z)\gamma^5 n(x_1)]$$
$$\times D^\dagger_\mu(x_2)F_{\alpha\beta}(x_3))\int\frac{d^4k_1}{\pi^2 i}\, e^{ik_1\cdot(x_1-z)}$$
$$\times{\rm tr}\{S^{\alpha\omega}(k_1+k_3)\Theta_{\omega\varphi}\frac{1}{M_{\rm N}-\hat{k}_1+\hat{k}_2}\gamma^\mu\frac{1}{M_{\rm N}-\hat{k}_1}\gamma^\beta\gamma^5\}$$
$$-{\rm T}([\bar{p^c}(x_1)\gamma_\nu\gamma^5 n(z) + \bar{p^c}(z)\gamma_\nu\gamma^5 n(x_1)]\, D^\dagger_\mu(x_2)$$
$$\times F^{\alpha\beta}(x_3))\int\frac{d^4k_1}{\pi^2 i}\, e^{ik_1\cdot(x_1-z)}\,{\rm tr}\{S^{\alpha\omega}(k_1+k_3)\,\Theta_{\omega\varphi}$$
$$\times\frac{1}{M_{\rm N}-\hat{k}_1+\hat{k}_2}\gamma^\mu\frac{1}{M_{\rm N}-\hat{k}_1}\gamma^\beta\gamma^5\}\Big\}. \qquad (4.5)$$

The effective Lagrangian eq. (4.5) defines the contribution of the $\Delta(1232)$ resonance to the low-energy transition n + p → D + γ.

The matrix element of the neutron-proton radiative capture caused by the contribution of the $\Delta(1232)$ resonance exchange is equal to

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to\Delta\mathrm{N}\to\mathrm{D}+\gamma)=$$
$$=-\frac{ie}{2M_{\mathrm{N}}^2}\frac{g_{\mathrm{V}}}{6\pi^2}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}$$
$$\times[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,(k_\alpha e^*_\beta(k)-k_\beta e^*_\alpha(k))\,e^*_\mu(k_{\mathrm{D}})$$
$$\times\,\mathcal{J}_5^{\mu\beta\alpha}(k_{\mathrm{D}},k)$$
$$+\frac{ie}{2M_{\mathrm{N}}^2}\frac{g_{\mathrm{V}}}{6\pi^2}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}$$
$$\times[\bar{u^c}(p_2)\gamma_\nu\gamma^5 u(p_1)]\,(k_\alpha e^*_\beta(k)-k_\beta e^*_\alpha(k))\,e^*_\mu(k_{\mathrm{D}})$$
$$\times\,\mathcal{J}_5^{\nu\mu\beta\alpha}(k_{\mathrm{D}},k),\qquad(4.6)$$

where the structure functions $\mathcal{J}_5^{\mu\beta\alpha}(k_{\mathrm{D}},k)$ and $\mathcal{J}_5^{\nu\mu\beta\alpha}(k_{\mathrm{D}},k)$ are defined by the momentum integrals

$$\mathcal{J}_5^{\mu\beta\alpha}(k_{\mathrm{D}},k)=$$
$$=\int\frac{\mathrm{d}^4k_1}{\pi^2 i}\,\mathrm{tr}\{(k_1+k_{\mathrm{D}})^\varphi S^{\alpha\omega}(k_1+k)\,\Theta_{\omega\varphi}$$
$$\times\frac{1}{M_{\mathrm{N}}-\hat{k}_1+\hat{k}_{\mathrm{D}}}\gamma^\mu\frac{1}{M_{\mathrm{N}}-\hat{k}_1}\gamma^\beta\gamma^5\},$$
$$\mathcal{J}_5^{\nu\mu\beta\alpha}(k_{\mathrm{D}},k)=$$
$$=\int\frac{\mathrm{d}^4k_1}{\pi^2 i}\,\mathrm{tr}\{(k_1+k_{\mathrm{D}})^\varphi S^{\alpha\omega}(k_1+k)\,\Theta_{\omega\varphi}\,\gamma^\nu$$
$$\times\frac{1}{M_{\mathrm{N}}-\hat{k}_1+\hat{k}_{\mathrm{D}}}\gamma^\mu\frac{1}{M_{\mathrm{N}}-\hat{k}_1}\gamma^\beta\gamma^5\}.\qquad(4.7)$$

At leading order in the large N_{C} expansion the structure functions eq. (4.7) read

$$\mathcal{J}_5^{\mu\beta\alpha}(k_{\mathrm{D}},k)=\frac{4}{3}\left(Z-\frac{1}{2}\right)i\,M_{\mathrm{N}}\,\varepsilon^{\mu\beta\alpha\lambda}\,k_{\mathrm{D}\lambda},$$
$$\mathcal{J}_5^{\nu\mu\beta\alpha}(k_{\mathrm{D}},k)=\frac{2}{3}\left(Z-\frac{1}{2}\right)i\,M^2_{\mathrm{N}}\,\varepsilon^{\mu\beta\alpha\nu}.\qquad(4.8)$$

We have neglected the mass difference between the masses of the $\Delta(1232)$ resonance and the nucleon. The matrix element of the low-energy transition $\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma$ caused by the $\Delta(1232)$ resonance contribution is equal to

$$\mathcal{M}_\Delta(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)=$$
$$\frac{e}{2M_{\mathrm{N}}}\frac{g_{\mathrm{V}}}{4\pi^2}\left[\left(\frac{1}{2}-Z\right)\frac{8}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}\right]\varepsilon^{\alpha\beta\mu\nu}k_\alpha$$
$$\times e^*_\beta(k,\lambda)e^*_\mu(k_{\mathrm{D}},\lambda_{\mathrm{D}})[\bar{u^c}(p_2)(2k_{\mathrm{D}\nu}-M_{\mathrm{N}}\gamma_\nu)\gamma^5u(p_1)]=$$
$$=e\,\frac{5g_{\mathrm{V}}}{8\pi^2}\left[\left(\frac{1}{2}-Z\right)\frac{8}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}\right]$$
$$\times(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})[\bar{u^c}(p_2)\gamma^5u(p_1)].\qquad(4.9)$$

In turn, the contribution of the nucleon tensor current [1]

$$\delta\mathcal{L}_{\mathrm{npD}}(x)=$$
$$\frac{g_{\mathrm{T}}}{2M_{\mathrm{N}}}\,[\bar{p^c}(x)\sigma^{\mu\nu}n(x)-\bar{n^c}(x)\sigma^{\mu\nu}p(x)]\,D^\dagger_{\mu\nu}(x)\qquad(4.10)$$

does not depend on the parameter Z and reads

$$\delta\mathcal{M}_\Delta(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)=$$
$$e\,\frac{5g_{\mathrm{V}}}{8\pi^2}\left[\frac{1}{5}\frac{g_{\mathrm{T}}}{g_{\mathrm{V}}}\frac{8}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}\right]$$
$$\times(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,[\bar{u^c}(p_2)\gamma^5u(p_1)].\qquad(4.11)$$

The coupling constants g_{T} and g_{V} are connected by the relation [1]

$$g_{\mathrm{T}}=\sqrt{\frac{3}{8}}\,g_{\mathrm{V}}+O(1/\sqrt{N_{\mathrm{C}}}).\qquad(4.12)$$

The total amplitude of the neutron-proton radiative capture for thermal neutrons reads

$$\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)=e\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\frac{5g_{\mathrm{V}}}{8\pi^2}C_{\mathrm{NN}}$$
$$\times(\mathbf{k}\times\mathbf{e}^*(\mathbf{k},\lambda))\cdot\mathbf{e}^*(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,[\bar{u^c}(p_2)\gamma^5u(p_1)]$$
$$\times\left[1+\frac{g^2_{\pi\mathrm{NN}}}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{M_\pi^2}{8\pi^2}\frac{\alpha_\rho}{\pi}\left(J_{\pi\mathrm{a_1N}}+\frac{3}{2g_{\mathrm{A}}}\,J_{\pi\mathrm{VN}}\right)\right.$$
$$+\frac{1}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{1}{5}\sqrt{\frac{3}{8}}\frac{1}{C_{\mathrm{NN}}}\frac{8}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}$$
$$\left.+\frac{1-2Z}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{1}{C_{\mathrm{NN}}}\frac{4}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}\right].\qquad(4.13)$$

The total cross-section for the neutron-proton radiative capture is then defined by

$$\sigma(\mathrm{np}\to\mathrm{D}\gamma)(p)=\frac{1}{v}\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})^2\,\frac{25}{64}\,\frac{\alpha}{\pi^2}\,Q_{\mathrm{D}}\,C^2_{\mathrm{NN}}\,M_{\mathrm{N}}\,\varepsilon^3_{\mathrm{D}}$$
$$\times\left[1+\frac{g^2_{\pi\mathrm{NN}}}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{M_\pi^2}{8\pi^2}\frac{\alpha_\rho}{\pi}\left(J_{\pi\mathrm{a_1N}}+\frac{3}{2g_{\mathrm{A}}}\,J_{\pi\mathrm{VN}}\right)\right.$$
$$+\frac{1}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{1}{5}\sqrt{\frac{3}{8}}\frac{1}{C_{\mathrm{NN}}}\frac{8}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}$$
$$\left.+\frac{1-2Z}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\frac{1}{C_{\mathrm{NN}}}\frac{4}{9}\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\frac{g^3_{\pi\mathrm{NN}}}{4M_\pi^2}\right]^2.\qquad(4.14)$$

The numerical value of the cross-section amounts to

$$\sigma(\mathrm{np}\to\mathrm{D}\gamma)(T_{\mathrm{n}})=325.5\,(1+0.246\,(1-2Z))^2\ \mathrm{mb}.\qquad(4.15)$$

Thus, the discrepancy of the theoretical cross-section and the experimental value eq. (1.1) can by described by the contribution of the $\Delta(1232)$ resonance. In order to fit the experimental value of the cross-section we should set $Z=0.473$. This agrees with the experimental bound $|Z|\le 1/2$ [18]. At $Z=1/2$ that is favoured theoretically [15] we get the cross-section $\sigma(\mathrm{np}\to\mathrm{D}\gamma)(T_{\mathrm{n}})=325.5\,\mathrm{mb}$ agreeing with the experimental value with accuracy better than 3%.

5 The photomagnetic disintegration of the deuteron

The amplitude of the photomagnetic disintegration of the deuteron $\gamma+\mathrm{D}\to\mathrm{n}+\mathrm{p}$ is related to the amplitude of the neutron-proton radiative capture n + p → D + γ due to time-reversal invariance and reads

$$\mathcal{M}(\gamma + \mathrm{D} \to \mathrm{n} + \mathrm{p}) = e\,(\mu_{\mathrm{p}} - \mu_{\mathrm{n}})$$
$$\frac{5g_{\mathrm{V}}}{8\pi^2}\,C_{\mathrm{NN}}\,(\mathbf{k}\times\mathbf{e}(\mathbf{k},\lambda))\cdot\mathbf{e}(\mathbf{k}_{\mathrm{D}},\lambda_{\mathrm{D}})\,[\bar{u}(p_2)\gamma^5 u^c(p_1)]$$
$$\times\Bigg[1 + \frac{g^2_{\pi\mathrm{NN}}}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{M^2_{\pi}}{8\pi^2}\,\frac{\alpha_{\rho}}{\pi}\left(J_{\pi\mathrm{a_1N}} + \frac{3}{2g_{\mathrm{A}}}\,J_{\pi\mathrm{VN}}\right)$$
$$+\frac{1}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{1}{5}\sqrt{\frac{3}{8}}\,\frac{1}{C_{\mathrm{NN}}}\,\frac{8}{9}\,\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g^3_{\pi\mathrm{NN}}}{4M^2_{\pi}}$$
$$+\frac{1-2Z}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{1}{C_{\mathrm{NN}}}\,\frac{4}{9}\,\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g^3_{\pi\mathrm{NN}}}{4M^2_{\pi}}\Bigg]^2$$
$$\times e^{i\,\delta_{\mathrm{np}}(p)}\,\frac{\sin\delta_{\mathrm{np}}(p)}{-a_{\mathrm{np}}\,p}\,\frac{1}{1+r^2_{\mathrm{D}}p^2}. \qquad (5.1)$$

The cross-section defined by the amplitude eq. (5.1) is then given by

$$\sigma(\gamma\mathrm{D}\to\mathrm{np})(\omega) = \sigma_0\left(\frac{\omega}{\varepsilon_{\mathrm{D}}}\right)$$
$$\times\frac{1}{\left(1-\dfrac{1}{2}r_{\mathrm{np}}a_{\mathrm{np}}p^2\right)^2 + a^2_{\mathrm{np}}p^2}\,\frac{r_{\mathrm{D}}p}{(1+r^2_{\mathrm{D}}p^2)^2}, \qquad (5.2)$$

where $p = \sqrt{M_{\mathrm{N}}(\omega-\varepsilon_{\mathrm{D}})}$ is the relative momentum of the np pair in the ${}^1S_0$ state, ω is the energy of the photon, and σ_0 is equal to

$$\sigma_0 = (\mu_{\mathrm{p}}-\mu_{\mathrm{n}})^2\,\frac{25\alpha Q_{\mathrm{D}}}{192\pi^2}\,C^2_{\mathrm{NN}}\,\varepsilon^{3/2}_{\mathrm{D}}M^{5/2}_{\mathrm{N}}$$
$$\times\Bigg[1 + \frac{g^2_{\pi\mathrm{NN}}}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{M^2_{\pi}}{8\pi^2}\,\frac{\alpha_{\rho}}{\pi}\left(J_{\pi\mathrm{a_1N}} + \frac{3}{2g_{\mathrm{A}}}\,J_{\pi\mathrm{VN}}\right)$$
$$+\frac{1}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{1}{5}\sqrt{\frac{3}{8}}\,\frac{1}{C_{\mathrm{NN}}}\,\frac{8}{9}\,\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g^3_{\pi\mathrm{NN}}}{4M^2_{\pi}}$$
$$+\frac{1-2Z}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\,\frac{1}{C_{\mathrm{NN}}}\,\frac{4}{9}\,\frac{g_{\pi\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g_{\gamma\mathrm{N}\Delta}}{g_{\pi\mathrm{NN}}}\,\frac{g^3_{\pi\mathrm{NN}}}{4M^2_{\pi}}\Bigg]^2 = 7.10\,\mathrm{mb}. \qquad (5.3)$$

The energy region of the dominance of the photomagnetic disintegration of the deuteron is restricted by the constraint $r_{\mathrm{D}}p = \sqrt{(\omega-\varepsilon_{\mathrm{D}})/\varepsilon_{\mathrm{D}}} \ll 1$ or differently $\omega \gtrsim \varepsilon_{\mathrm{D}} = 2.225\,\mathrm{MeV}$.

The numerical values of the cross-section for the photomagnetic disintegration of the deuteron at energies $\omega \le 2\,\varepsilon_{\mathrm{D}} = 4.45\,\mathrm{MeV}$ read

$$\sigma(\gamma\mathrm{D}\to\mathrm{np})(\omega)\Big|_{\omega=2.62\,\mathrm{MeV}} = 0.358\,(0.380)\,\mathrm{mb},$$
$$\sigma(\gamma\mathrm{D}\to\mathrm{np})(\omega)\Big|_{\omega=2.76\,\mathrm{MeV}} = 0.302\,(0.327)\,\mathrm{mb},$$
$$\sigma(\gamma\mathrm{D}\to\mathrm{np})(\omega)\Big|_{\omega=4.45\,\mathrm{MeV}} = 0.094\,(0.128)\,\mathrm{mb}, \qquad (5.4)$$

where in parentheses we have adduced the theoretical values obtained by Chen and Savage in the EFT [10]. One can see a reasonable agreement between the results obtained in the NNJL model and the EFT. Thus, the spatial smearing of the physical deuteron caused by the effective radius r_{D} and introduced in the NNJL model phenomenologically in the form of the wave function eq. (2.16) describes well the energy dependence of the cross-section for the photomagnetic disintegration of the deuteron at photon energies far from threshold. Note that in the critical region of the photon energies $\omega \le 2\,\varepsilon_{\mathrm{D}} = 4.45\,\mathrm{MeV}$ the cross-section for the photomagnetic disintegration of the deuteron calculated in the NNJL model falls steeper with ω than in the EFT. However, in this energy region the dominant role is attributed to the E1-transition which we have not taken into account.

For the correct description of the experimental data on the photodisintegration of the deuteron [40] (see also Chen and Savage [10]):

$$\sigma(\gamma\mathrm{D}\to\mathrm{np})_{\exp}(\omega)\Big|_{\omega=2.62\,\mathrm{MeV}} = (1.300\pm 0.029)\,\mathrm{mb},$$
$$\sigma(\gamma\mathrm{D}\to\mathrm{np})_{\exp}(\omega)\Big|_{\omega=2.76\,\mathrm{MeV}} = (1.474\pm 0.032)\,\mathrm{mb},$$
$$\sigma(\gamma\mathrm{D}\to\mathrm{np})_{\exp}(\omega)\Big|_{\omega=4.45\,\mathrm{MeV}} = (2.430\pm 0.170)\,\mathrm{mb} \qquad (5.5)$$

one must include the contribution of the E1-transition [10].

Since the cross-section for the photodisintegration of the deuteron having been discussed above does not contain the contribution of the E1-transition, our result can be scarcely compared with the recent theoretical investigation of the photodisintegration of the deuteron carried out by Anisovich and Sadovnikova [41] within the dispersion relation approach based on the dispersion relation technique developed by Anisovich *et al.* [42]. This investigation represents a detailed analysis of the saturation of the cross-section for the photodisintegration of the deuteron by different intermediate states valid mainly for the photon energy region $\omega \ge 50\,\mathrm{MeV}$, where the contribution of the E1-transition is important.

6 The amplitude of the solar proton burning

The analysis of the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{D}+\mathrm{e}^+ + \nu_{\mathrm{e}}$ within the NNJL model we should start with the evaluation of the effective Lagrangian $\mathcal{L}^{\mathrm{pp}\to\mathrm{De}^+\nu_{\mathrm{e}}}_{\mathrm{eff}}(x)$ of the transition $\mathrm{p}+\mathrm{p}\to\mathrm{D}+\mathrm{e}^+ + \nu_{\mathrm{e}}$. In the tree-meson approximation[1] the effective Lagrangian $\mathcal{L}^{\mathrm{pp}\to\mathrm{De}^+\nu_{\mathrm{e}}}_{\mathrm{eff}}(x)$ is defined by the one-nucleon loop diagrams depicted in fig. 3. The detailed evaluation of $\mathcal{L}^{\mathrm{pp}\to\mathrm{De}^+\nu_{\mathrm{e}}}_{\mathrm{eff}}(x)$ is given in appendix.

[1] Below the analysis of weak nuclear reactions is carried out in the tree-meson approximation. The inclusion of chiral one-meson loop corrections goes beyond the scope of this paper and demands a separate publication.

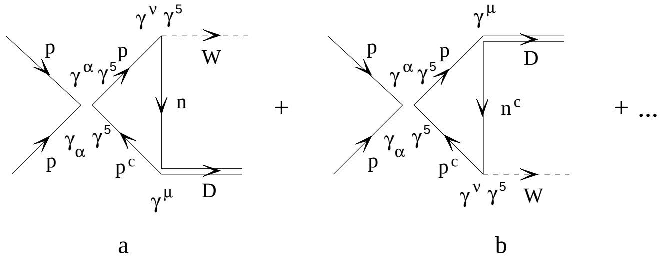

Fig. 3. One-nucleon loop diagrams describing the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$ of the low-energy transition p + p → D + e$^+$ + $\nu_{\rm e}$.

The result reads[2]

$$\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x) = -ig_{\rm A}C_{\rm NN}M_{\rm N}\frac{G_{\rm V}}{\sqrt{2}}\frac{3g_{\rm V}}{4\pi^2}\,D^\dagger_\mu(x)$$
$$\times[\bar{p^c}(x)\gamma^5 p(x)]\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\mu(1-\gamma^5)\psi_{\rm e}(x)], \qquad (6.1)$$

where $G_{\rm V} = G_{\rm F}\cos\vartheta_{\rm C}$ with $G_{\rm F} = 1.166\times 10^{-11}\,{\rm MeV}^{-2}$ and $\vartheta_{\rm C}$ are the Fermi weak-coupling constant and the Cabibbo angle $\cos\vartheta_{\rm C} = 0.975$.

For the derivation of the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$ we have used the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm NN\to NN}(x)$ responsible for low-energy transitions N + N → N + N defined by eq. (1.17).

The matrix element of the transition p + p → D + e$^+$ + $\nu_{\rm e}$ we define in the usual way

$$\int d^4x\,\langle D(k_{\rm D})e^+(k_{\rm e^+})\nu_{\rm e}(k_{\nu_{\rm e}})|\mathcal{L}_{\rm eff}^{\rm np\to De^+\nu_e}(x)|p(p_1)p(p_2)\rangle =$$
$$(2\pi)^4\delta^{(4)}(k_{\rm D}+k_{\rm e^+}+k_{\nu_{\rm e}}-p_1-p_2)$$
$$\times\frac{\mathcal{M}({\rm n+p\to D+e^++\nu_e})}{\sqrt{2E_1V\,2E_2V\,2E_{\rm D}V\,2E_{\rm e^+}V\,2E_{\nu_{\rm e}}V}}, \qquad (6.2)$$

where $E_i\,(i=1,2,{\rm D},{\rm e}^+,\nu_{\rm e})$ are the energies of the protons, the deuteron, the positron and the neutrino, V is the normalization volume.

The wave functions of the initial $|p(p_1)p(p_2)\rangle$ and final $\langle D(k_{\rm D})e^+(k_{\rm e^+})\nu_{\rm e}(k_{\nu_{\rm e}})|$ state we take in the usual form

$$|p(p_1)p(p_2)\rangle = \frac{1}{\sqrt{2}}\,a^\dagger_{\rm p}(\mathbf{p}_1,\sigma_1)\,a^\dagger_{\rm p}(\mathbf{p}_2,\sigma_2)|0\rangle,$$
$$\langle D(k_{\rm D})e^+(k_{\rm e^+})\nu_{\rm e}(k_{\nu_{\rm e}})| =$$
$$\langle 0|a_{\rm D}(\mathbf{k}_{\rm D},\lambda_{\rm D})\,a_{\rm e^+}(\mathbf{k}_{\rm e^+},\sigma)\,a_{\nu_{\rm e}}(\mathbf{k}_{\nu_{\rm e}},\sigma'), \qquad (6.3)$$

where $a^\dagger_{\rm n}(\mathbf{p}_1,\sigma_1)$ and $a^\dagger_{\rm p}(\mathbf{p}_2,\sigma_2)$ are the operators of creation of the protons. In turn, $a_{\rm D}(\mathbf{k}_{\rm D},\lambda_{\rm D})$, $a_{\rm e^+}(\mathbf{k}_{\rm e^+},\sigma_{\rm e^+})$ and $a_{\nu_{\rm e}}(\mathbf{k}_{\nu_{\rm e}},\sigma_{\nu_{\rm e}})$ are the operators of annihilation of the deuteron, the positron and the neutrino.

The effective Lagrangian eq. (6.1) defines the effective vertex of the low-energy nuclear transition p + p → D + e$^+$ + $\nu_{\rm e}$

$$i\mathcal{M}({\rm p+p\to D+e^++\nu_e}) = g_{\rm A}M_{\rm N}\,C_{\rm NN}\,G_{\rm V}\,\frac{3g_{\rm V}}{4\pi^2}\,e^*_\mu(k_{\rm D})$$
$$\times[\bar{u}(k_{\nu_{\rm e}})\gamma^\mu(1-\gamma^5)v(k_{\rm e^+})][\bar{u^c}(p_2)\gamma^5 u(p_1)], \qquad (6.4)$$

where $e^*_\mu(k_{\rm D},\lambda_{\rm D})$ is a 4-vector of a polarization of the deuteron, $u(k_{\nu_{\rm e}})$, $v(k_{\rm e^+})$, $u(p_2)$ and $u(p_1)$ are the Dirac bispinors of the neutrino, the positron and the protons, respectively.

For the evaluation of the matrix element eq. (6.4) we have used the wave functions of the protons in the form of the plane waves. However, a real wave function of the pp pair in the $^1S_0$ state is defined by low-energy nuclear forces and Coloumb repulsion. In order to take into account both low-energy nuclear forces and Coulomb repulsion for the relative movement of the pp pair in the $^1S_0$ state we would sum up an infinite series of one-proton bubbles. The vertices of these one-nucleon bubbles are defined by

$$V_{\rm pp\to pp}(k',k) = C_{\rm NN}\,\psi^*_{\rm pp}(k')$$
$$\times[\bar{u}(p'_2)\gamma^5 u^c(p'_1)]\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,\psi_{\rm pp}(k), \qquad (6.5)$$

where $\psi_{\rm pp}(k)$ and $\psi^*_{\rm pp}(k')$ are the explicit Coulomb wave functions of the relative movement of the protons taken at zero relative distances, and k and k' are relative 3-momenta of the protons $\mathbf{k} = (\mathbf{p}_1-\mathbf{p}_2)/2$ and $\mathbf{k}' = (\mathbf{p}'_1-\mathbf{p}'_2)/2$ in the initial and final states. The explicit form of $\psi_{\rm pp}(k)$ we take following Kong and Ravndal [33] (see also [43]):

$$\psi_{\rm pp}(k) = e^{-\pi/4kr_{\rm C}}\,\Gamma\Big(1+\frac{i}{2kr_{\rm C}}\Big), \qquad (6.6)$$

where $2\,r_{\rm C} = 2/M_{\rm N}\alpha = 57.64\,{\rm fm}$ and $\alpha = 1/137$ are the Bohr radius of the pp pair in the $^1S_0$ state and the fine-structure constant. The squared value of the modulo of $\psi_{\rm pp}(k)$ is given by

$$|\psi_{\rm pp}(k)|^2 = C_0^2(k) = \frac{\pi}{kr_{\rm C}}\,\frac{1}{e^{\pi/kr_{\rm C}}-1}, \qquad (6.7)$$

where $C_0(k)$ is the Gamow penetration factor [3,29,43].

By taking into account the contribution of the Coulomb wave function and summing up an infinite series of one-proton bubbles the expression eq. (6.4) can be recast into the form

see equation (6.8) *on next page*

where $P = p_1+p_2 = (2\sqrt{k^2+M_{\rm N}^2},\mathbf{0})$ is the 4-momentum of the pp-pair in the center-of-mass frame; $Q = a\,P + b\,K = a\,(p_1+p_2) + b\,(p_1-p_2)$ is an arbitrary shift of virtual momentum with arbitrary parameters a and b, and in the center of mass frame $K = p_1-p_2 = (0,2\,\mathbf{k})$. The parameters a and b can be functions of k.

The evaluation of the momentum integral we would carry out at leading order in the $1/M_{\rm N}$ expansion or differently in the large $N_{\rm C}$ expansion [1] due to proportionality $M_{\rm N}\sim N_{\rm C}$ valid in QCD with $SU(N_{\rm C})$ gauge group

[2] This result is obtained at zero contribution of the nucleon tensor current (see appendix and discussion in the conclusion).

$$i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^{+}+\nu_e) = g_{\mathrm{A}} M_{\mathrm{N}}\, C_{\mathrm{NN}}\, G_{\mathrm{V}}\, \frac{3g_{\mathrm{V}}}{4\pi^2}\, e^{*}_{\mu}(k_{\mathrm{D}})\, [\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})] \times \frac{[\bar{u^c}(p_2)\gamma^5 u(p_1)]\,\psi_{\mathrm{pp}}(k)}{1+\dfrac{C_{\mathrm{NN}}}{16\pi^2}\displaystyle\int \frac{d^4p}{\pi^2 i}\,|\psi_{\mathrm{pp}}(|\mathbf{p}+\mathbf{Q}\,|)|^2 \mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{Q}}\Big\}}. \quad (6.8)$$

at $N_{\mathrm{C}}\to\infty$ [22]. As a result we obtain

$$\int \frac{\mathrm{d}^4p}{\pi^2 i}\,|\psi_{\mathrm{pp}}(|\mathbf{p}+\mathbf{Q}\,|)|^2\, \mathrm{tr}\Big\{\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{P}-\hat{Q}}\gamma^5\frac{1}{M_{\mathrm{N}}-\hat{p}-\hat{Q}}\Big\} = -8\,a\,(a+1)\,M_{\mathrm{N}}^2 +8\,(b^2-a\,(a+1))\,k^2 - i\,8\pi\,M_{\mathrm{N}}\,k\,|\psi_{\mathrm{pp}}(k)|^2 = -8\,a\,(a+1)\,M_{\mathrm{N}}^2 +8\,(b^2-a\,(a+1))\,k^2 - i\,8\pi\,M_{\mathrm{N}}\,k\,C_0^2(k). \quad (6.9)$$

Substituting eq. (6.9) in eq. (6.8), we get

$$i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^{+}+\nu_e) = g_{\mathrm{A}} M_{\mathrm{N}}\, C_{\mathrm{NN}}\, G_{\mathrm{V}}\, \frac{3g_{\mathrm{V}}}{4\pi^2} \times e^{*}_{\mu}(k_{\mathrm{D}})\, [\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})] \times[\bar{u^c}(p_2)\gamma^5 u(p_1)]\, e^{-\pi/4kr_{\mathrm{C}}}\,\Gamma\Big(1+\frac{i}{2kr_{\mathrm{C}}}\Big) \times\Big[1-a(a+1)\frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2+\frac{C_{\mathrm{NN}}}{2\pi^2} \times(b^2-a\,(a+1))\,k^2 - i\,\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C_0^2(k)\Big]^{-1}. \quad (6.10)$$

In order to reconcile the contribution of low-energy elastic pp scattering with low-energy nuclear phenomenology [43] we should make a few changes. To this aim we should rewrite eq. (6.10) in a more convenient form:

$$i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^{+}+\nu_e) = g_{\mathrm{A}} M_{\mathrm{N}}\, C_{\mathrm{NN}}\, G_{\mathrm{V}}\, \frac{3g_{\mathrm{V}}}{4\pi^2} \times e^{*}_{\mu}(k_{\mathrm{D}})\, [\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})] \times[\bar{u^c}(p_2)\gamma^5 u(p_1)]\, e^{i\sigma_0(k)}\,C_0(k) \times\Big[1-a(a+1)\frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2+\frac{C_{\mathrm{NN}}}{2\pi^2} \times(b^2-a\,(a+1))\,k^2 - i\,\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C_0^2(k)\Big]^{-1}. \quad (6.11)$$

We have denoted

$$e^{-\pi/4kr_{\mathrm{C}}}\,\Gamma\Big(1+\frac{i}{2kr_{\mathrm{C}}}\Big) = e^{i\sigma_0(k)}\,C_0(k)\,, \qquad \sigma_0(k) = \arg\Gamma\Big(1+\frac{i}{2kr_{\mathrm{C}}}\Big), \quad (6.12)$$

where $\sigma_0(k)$ is a pure Coulomb phase shift.

Now, let us rewrite the denominator of the amplitude eq. (6.11) in the equivalent form

$$\Big\{\cos\sigma_0(k)\Big[1-a(a+1)\frac{C_{\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2+\frac{C_{\mathrm{NN}}}{2\pi^2}\,(b^2-a\,(a+1))\,k^2\Big] -\sin\sigma_0(k)\,\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C_0^2(k)\Big\} - i\,\Big\{\cos\sigma_0(k)\,\frac{G_{\pi\mathrm{NN}}M_{\mathrm{N}}}{2\pi} \times k\,C_0^2(k)+\sin\sigma_0(k)\Big[1-a(a+1)\frac{C_{\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2 +\frac{C_{\mathrm{NN}}}{2\pi^2}\,(b^2-a\,(a+1))\,k^2\Big]\Big\} = \frac{1}{\mathcal{Z}}\Big[1-\frac{1}{2}\,a^{\mathrm{e}}_{\mathrm{pp}}r^{\mathrm{e}}_{\mathrm{pp}}k^2+\frac{a^{\mathrm{e}}_{\mathrm{pp}}}{r_{\mathrm{C}}}\,h(2kr_{\mathrm{C}})+i\,a^{\mathrm{e}}_{\mathrm{pp}}\,k\,C_0^2(k)\Big], \quad (6.13)$$

where we have denoted

$$\frac{1}{\mathcal{Z}}\Big[1-\frac{1}{2}\,a^{\mathrm{e}}_{\mathrm{pp}}r^{\mathrm{e}}_{\mathrm{pp}}k^2+\frac{a^{\mathrm{e}}_{\mathrm{pp}}}{r_{\mathrm{C}}}\,h(2kr_{\mathrm{C}})\Big] = -\sin\sigma_0(k)\,\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C_0^2(k)+\cos\sigma_0(k) \times\Big[1-a(a+1)\frac{G_{\pi\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2+\frac{C_{\mathrm{NN}}}{2\pi^2}\,(b^2-a\,(a+1))\,k^2\Big],$$
$$-\frac{1}{\mathcal{Z}}\,a^{\mathrm{e}}_{\mathrm{pp}}\,k\,C_0^2(k) = \cos\sigma_0(k)\frac{C_{\mathrm{NN}}M_{\mathrm{N}}}{2\pi}\,k\,C_0^2(k)+\sin\sigma_0(k) \times\Big[1-a(a+1)\frac{C_{\mathrm{NN}}}{2\pi^2}\,M_{\mathrm{N}}^2+\frac{C_{\mathrm{NN}}}{2\pi^2}\,(b^2-a\,(a+1))\,k^2\Big]. \quad (6.14)$$

Here $\mathcal{Z}$ is a constant which we would remove by the renormalization of the wave functions of the protons, $a^{\mathrm{e}}_{\mathrm{pp}} = (-7.8196\pm 0.0026)\,\mathrm{fm}$ and $r^{\mathrm{e}}_{\mathrm{pp}} = 2.790\pm 0.014\,\mathrm{fm}$ [44] are the S wave scattering length and the effective range of pp scattering in the ${}^1S_0$ state with the Coulomb repulsion, and $h(2kr_{\mathrm{C}})$ is defined by [43]

$$h(2kr_{\mathrm{C}}) = -\gamma+\ln(2kr_{\mathrm{C}})+\sum^{\infty}_{n=1}\frac{1}{n(1+4n^2k^2r^2_{\mathrm{C}})}. \quad (6.15)$$

The validity of the relations eq. (6.14) assumes the dependence of parameters a and b on the relative momentum k.

After the changes eq. (6.13) and eq. (6.14) the amplitude eq. (6.11) takes the form

$$i\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+\mathrm{e}^{+}+\nu_e) = G_{\mathrm{V}}\,g_{\mathrm{A}} M_{\mathrm{N}}\, C_{\mathrm{NN}}\, G_{\mathrm{V}}\, \frac{3g_{\mathrm{V}}}{4\pi^2} \times e^{*}_{\mu}(k_{\mathrm{D}})\, [\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^{\mu}(1-\gamma^5)v(k_{\mathrm{e}^+})] \times\frac{C_0(k)}{1-\dfrac{1}{2}\,a^{\mathrm{e}}_{\mathrm{pp}}r^{\mathrm{e}}_{\mathrm{pp}}k^2+\dfrac{a^{\mathrm{e}}_{\mathrm{pp}}}{r_{\mathrm{C}}}\,h(2kr_{\mathrm{C}})+i\,a^{\mathrm{e}}_{\mathrm{pp}}\,k\,C_0^2(k)} \times\mathcal{Z}\,[\bar{u^c}(p_2)\gamma^5 u(p_1)]. \quad (6.16)$$

Renormalizing the wave functions of the protons $\sqrt{\mathcal{Z}}u(p_2) \to u(p_2)$ and $\sqrt{\mathcal{Z}}u(p_1) \to u(p_1)$ we obtain the amplitude of the solar proton burning

$$\begin{aligned} &i\mathcal{M}(\mathrm{p}+\mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_e) = \\ &g_{\mathrm{A}} M_{\mathrm{N}}\, C_{\mathrm{NN}}\, G_{\mathrm{V}}\, \frac{3g_{\mathrm{V}}}{4\pi^2}\, e^*_\mu(k_{\mathrm{D}})\, [\bar{u}(k_{\nu_{\mathrm{e}}})\gamma^\mu(1-\gamma^5)v(k_{\mathrm{e}^+})] \\ &\times \frac{C_0(k)}{1 - \frac{1}{2}\, a^{\mathrm{e}}_{\mathrm{pp}} r^{\mathrm{e}}_{\mathrm{pp}} k^2 + \frac{a^{\mathrm{e}}_{\mathrm{pp}}}{r_{\mathrm{C}}}\, h(2kr_{\mathrm{C}}) + i\, a^{\mathrm{e}}_{\mathrm{pp}}\, k\, C_0^2(k)} \\ &\times [\bar{u^c}(p_2)\gamma^5 u(p_1)]\, F_{\mathrm{D}}(k^2), \qquad (6.17) \end{aligned}$$

where $F_{\mathrm{D}}(k^2)$ is given by eq. (2.16) and describes the spatial smearing of the deuteron coupled to the pp pair in the ${}^1S_0$ state.

The real part of the denominator of the amplitude eq. (6.17) is in complete agreement with a phenomenological relation [43]

$$\mathrm{ctg}\delta^{\mathrm{e}}_{\mathrm{pp}}(k) = \frac{1}{C_0^2(k)\, k}\Big[-\frac{1}{a^{\mathrm{e}}_{\mathrm{pp}}} + \frac{1}{2}\, r^{\mathrm{e}}_{\mathrm{pp}} k^2 - \frac{1}{r_{\mathrm{C}}}\, h(2kr_{\mathrm{C}})\Big], \qquad (6.18)$$

describing the phase shift $\delta^{\mathrm{e}}_{\mathrm{pp}}(k)$ of low-energy elastic pp scattering in terms of the S wave scattering length $a^{\mathrm{e}}_{\mathrm{pp}}$ and the effective range $r^{\mathrm{e}}_{\mathrm{pp}}$. Thus, we argue that the contribution of low-energy elastic pp scattering to the amplitude of the solar proton burning is described in agreement with low-energy nuclear phenomenology in terms of the S wave scattering length $a^{\mathrm{e}}_{\mathrm{pp}}$ and the effective range $r^{\mathrm{e}}_{\mathrm{pp}}$ taken from the experimental data [44].

7 The astrophysical factor for the solar proton burning

The amplitude eq. (6.17) squared, averaged over polarizations of the protons and summed over polarizations of final particles reads

$$\begin{aligned} &\overline{|\mathcal{M}(\mathrm{p}+\mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_e)|^2} = \\ &G^2_{\mathrm{V}}\, g^2_{\mathrm{A}} M^6_{\mathrm{N}}\, C^2_{\mathrm{NN}}\, \frac{54 Q_{\mathrm{D}}}{\pi^2}\, F^2_{\mathrm{D}}(k^2) \\ &\times \frac{C_0^2(k)}{\Big[1 - \frac{1}{2}\, a^{\mathrm{e}}_{\mathrm{pp}} r^{\mathrm{e}}_{\mathrm{pp}} k^2 + \frac{a^{\mathrm{e}}_{\mathrm{pp}}}{r_{\mathrm{C}}}\, h(2kr_{\mathrm{C}})\Big]^2 + (a^{\mathrm{e}}_{\mathrm{pp}})^2 k^2 C_0^4(k)} \\ &\times \Big(E_{\mathrm{e}^+} E_{\nu_{\mathrm{e}}} - \frac{1}{3}\mathbf{k}_{\mathrm{e}^+}\cdot \mathbf{k}_{\nu_{\mathrm{e}}}\Big), \qquad (7.1) \end{aligned}$$

where $m_{\mathrm{e}} = 0.511\,\mathrm{MeV}$ is the positron mass, and we have used the relation $g^2_{\mathrm{V}} = 2\,\pi^2\, Q_{\mathrm{D}}\, M^2_{\mathrm{N}}$.

The cross-section for the reaction $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$ is defined by

$$\begin{aligned} &\sigma^{\mathrm{pp}\to \mathrm{De}^+\nu_{\mathrm{e}}}(T_{\mathrm{pp}}) = \\ &\frac{1}{v}\,\frac{1}{4E_1E_2}\int \overline{|\mathcal{M}(\mathrm{p}+\mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_e)|^2} \\ &\times (2\pi)^4\, \delta^{(4)}(k_{\mathrm{D}} + k_\ell - p_1 - p_2) \\ &\times \frac{\mathrm{d}^3k_{\mathrm{D}}}{(2\pi)^3 2E_{\mathrm{D}}}\frac{\mathrm{d}^3k_{\mathrm{e}^+}}{(2\pi)^3 2E_{\mathrm{e}^+}}\frac{\mathrm{d}^3k_{\nu_{\mathrm{e}}}}{(2\pi)^3 2E_{\nu_{\mathrm{e}}}}, \qquad (7.2) \end{aligned}$$

where v is a relative velocity of the pp pair and $k_\ell = k_{\mathrm{e}^+} + k_{\nu_{\mathrm{e}}}$ is a 4-momentum of the leptonic pair.

The integration over the phase volume of the final $\mathrm{De}^+\nu_{\mathrm{e}}$ state we perform in the non-relativistic limit

$$\begin{aligned} &\int \frac{\mathrm{d}^3k_{\mathrm{D}}}{(2\pi)^3 2E_{\mathrm{D}}}\frac{\mathrm{d}^3k_{\mathrm{e}^+}}{(2\pi)^3 2E_{\mathrm{e}^+}}\frac{\mathrm{d}^3k_{\nu_{\mathrm{e}}}}{(2\pi)^3 2E_{\nu_{\mathrm{e}}}}\,(2\pi)^4 \\ &\times \delta^{(4)}(k_{\mathrm{D}} + k_\ell - p_1 - p_2)\Big(E_{\mathrm{e}^+}E_{\nu_{\mathrm{e}}} - \frac{1}{3}\mathbf{k}_{\mathrm{e}^+}\cdot\mathbf{k}_{\nu_{\mathrm{e}}}\Big) = \\ &\frac{1}{32\pi^3 M_{\mathrm{N}}}\int_{m_{\mathrm{e}}}^{W+T_{\mathrm{pp}}}\sqrt{E^2_{\mathrm{e}^+} - m^2_{\mathrm{e}}}\, E_{\mathrm{e}^+} \\ &\times (W + T_{\mathrm{pp}} - E_{\mathrm{e}^+})^2\,\mathrm{d}E_{\mathrm{e}^+} = \frac{(W+T_{\mathrm{pp}})^5}{960\pi^3 M_{\mathrm{N}}}\, f(\xi), \qquad (7.3) \end{aligned}$$

where $W = \varepsilon_{\mathrm{D}} - (M_{\mathrm{n}} - M_{\mathrm{p}}) = (2.225 - 1.293)\,\mathrm{MeV} = 0.932\,\mathrm{MeV}$ and $\xi = m_{\mathrm{e}}/(W + T_{\mathrm{pp}})$. The function $f(\xi)$ is defined by the integral

$$\begin{aligned} f(\xi) &= 30\int_\xi^1 \sqrt{x^2 - \xi^2}\, x\,(1-x)^2\mathrm{d}x = \\ &(1 - \frac{9}{2}\,\xi^2 - 4\,\xi^4)\,\sqrt{1-\xi^2} \\ &+ \frac{15}{2}\,\xi^4\ln\Big(\frac{1+\sqrt{1-\xi^2}}{\xi}\Big)\Big|_{T_{\mathrm{pp}}=0} = 0.222 \qquad (7.4) \end{aligned}$$

and normalized to unity at $\xi = 0$.

Thus, the cross-section for the solar proton burning is given by

see equation (7.5) *on next page*

The astrophysical factor $S_{\mathrm{pp}}(T_{\mathrm{pp}})$ reads

see equation (7.6) *on next page*

At zero kinetic energy of the relative movement of the protons $T_{\mathrm{pp}} = 0$ the astrophysical factor $S_{\mathrm{pp}}(0)$ is given by

$$\begin{aligned} S_{\mathrm{pp}}(0) &= \alpha\,\frac{9g^2_{\mathrm{A}} G^2_{\mathrm{V}} Q_{\mathrm{D}} M^4_{\mathrm{N}}}{1280\pi^4}\, C^2_{\mathrm{NN}}\, W^5\, f\Big(\frac{m_{\mathrm{e}}}{W}\Big) = \\ &4.08\,\times 10^{-25}\,\mathrm{MeV\,b}. \qquad (7.7) \end{aligned}$$

The value $S_{\mathrm{pp}}(0) = 4.08 \times 10^{-25}\,\mathrm{MeV\,b}$ agrees well with the recommended value $S_{\mathrm{pp}}(0) = 4.00 \times 10^{-25}\,\mathrm{MeV\,b}$ [29].

Unlike the astrophysical factor obtained by Kamionkowski and Bahcall [30], the astrophysical factor given by eq. (7.7) does not depend explicitly on the S wave scattering length of low-energy elastic pp scattering in the ${}^1S_0$ state. This is due to the normalization of the wave function of the pp pair. After the summation of an infinite series and by using the relation eq. (6.18) we obtain the wave function of the pp pair in the ${}^1S_0$ state in the form

$$\psi_{\mathrm{pp}}(k) = e^{i\,\delta^{\mathrm{e}}_{\mathrm{pp}}(k)}\,\frac{\sin\,\delta^{\mathrm{e}}_{\mathrm{pp}}(k)}{-a^{\mathrm{e}}_{\mathrm{pp}}kC_0(k)}, \qquad (7.8)$$

that corresponds the normalization of the wave function of the relative movement of the pp pair used by Schiavilla

$$\sigma^{\mathrm{pp}\to\mathrm{De}^+\nu_{\mathrm{e}}}(T_{\mathrm{pp}}) = \frac{e^{-\pi/r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}}}}{v^2}\,\alpha\,\frac{9g_{\mathrm{A}}^2G_{\mathrm{V}}^2Q_{\mathrm{D}}M_{\mathrm{N}}^3}{320\,\pi^4}\,C_{\mathrm{NN}}^2\,(W+T_{\mathrm{pp}})^5\,f\Big(\frac{m_{\mathrm{e}}}{W+T_{\mathrm{pp}}}\Big)$$
$$\times\frac{F_{\mathrm{D}}^2(M_{\mathrm{N}}T_{\mathrm{pp}})}{\Big[1-\frac{1}{2}\,a_{\mathrm{pp}}^{\mathrm{e}}r_{\mathrm{pp}}^{\mathrm{e}}M_{\mathrm{N}}T_{\mathrm{pp}}+\frac{a_{\mathrm{pp}}^{\mathrm{e}}}{r_{\mathrm{C}}}\,h(2r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}})\Big]^2+(a_{\mathrm{pp}}^{\mathrm{e}})^2M_{\mathrm{N}}T_{\mathrm{pp}}C_0^4(\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}})}$$
$$\times\frac{1}{1-e^{-\pi/r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}}}}=\frac{S_{\mathrm{pp}}(T_{\mathrm{pp}})}{T_{\mathrm{pp}}}\,e^{-\pi/r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}}}. \tag{7.5}$$

$$S_{\mathrm{pp}}(T_{\mathrm{pp}}) = \alpha\,\frac{9g_{\mathrm{A}}^2G_{\mathrm{V}}^2Q_{\mathrm{D}}M_{\mathrm{N}}^4}{1280\pi^4}\,\frac{C_{\mathrm{NN}}^2\,(W+T_{\mathrm{pp}})^5}{1-e^{-\pi/r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}}}}\,f\Big(\frac{m_{\mathrm{e}}}{W+T_{\mathrm{pp}}}\Big)$$
$$\times\frac{F_{\mathrm{D}}^2(M_{\mathrm{N}}T_{\mathrm{pp}})}{\Big[1-\frac{1}{2}\,a_{\mathrm{pp}}^{\mathrm{e}}r_{\mathrm{pp}}^{\mathrm{e}}M_{\mathrm{N}}T_{\mathrm{pp}}+\frac{a_{\mathrm{pp}}^{\mathrm{e}}}{r_{\mathrm{C}}}\,h(2r_{\mathrm{C}}\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}})\Big]^2+(a_{\mathrm{pp}}^{\mathrm{e}})^2M_{\mathrm{N}}T_{\mathrm{pp}}C_0^4(\sqrt{M_{\mathrm{N}}T_{\mathrm{pp}}})}. \tag{7.6}$$

et al. [31]. For the more detailed discussion of this problem we relegate readers to the paper by Schiavilla *et al.* [31] [3].

8 The reaction $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$

The evaluation of the amplitude of the reaction $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$ is analogous to that of the amplitude of the solar proton burning. The result reads

$$i\mathcal{M}(\nu_{\mathrm{e}}+\mathrm{D}\to\mathrm{e}^-+\mathrm{p}+\mathrm{p}) =$$
$$g_{\mathrm{A}}M_{\mathrm{N}}\frac{G_{\mathrm{V}}}{\sqrt{2}}\,\frac{3g_{\mathrm{V}}}{2\pi^2}\,C_{\mathrm{NN}}\,e^*_{\mu}(k_{\mathrm{D}})\,[\bar{u}(k_{\mathrm{e}^-})\gamma^{\mu}(1-\gamma^5)u(k_{\nu_{\mathrm{e}}})]$$
$$\times\frac{C_0(k)}{1-\frac{1}{2}\,a_{\mathrm{pp}}^{\mathrm{e}}r_{\mathrm{pp}}^{\mathrm{e}}k^2+\frac{a_{\mathrm{pp}}^{\mathrm{e}}}{r_{\mathrm{C}}}\,h(2kr_{\mathrm{C}})+i\,a_{\mathrm{pp}}^{\mathrm{e}}\,k\,C_0^2(k)}$$
$$\times[\bar{u}(p_2)\gamma^5u^c(p_1)]\,F_{\mathrm{D}}(k^2). \tag{8.1}$$

The amplitude eq. (8.1) squared, averaged over polarizations of the deuteron and summed over polarizations of the final particles reads

$$\overline{|\mathcal{M}(\nu_{\mathrm{e}}+\mathrm{D}\to\mathrm{e}^-+\mathrm{p}+\mathrm{p})|^2} =$$
$$S_{\mathrm{pp}}(0)\,\frac{2^{12}5\pi^2}{\Omega_{\mathrm{De}^+\nu_{\mathrm{e}}}}\,\frac{r_{\mathrm{C}}M_{\mathrm{N}}^3}{m_{\mathrm{e}}^5}\,F_{\mathrm{D}}^2(k^2)\,F_+(Z,E_{\mathrm{e}^-})$$
$$\times\frac{C_0^2(k)}{\Big[1-\frac{1}{2}a_{\mathrm{pp}}^{\mathrm{e}}r_{\mathrm{pp}}^{\mathrm{e}}k^2+\frac{a_{\mathrm{pp}}^{\mathrm{e}}}{r_{\mathrm{C}}}\,h(2kr_{\mathrm{C}})\Big]^2+(a_{\mathrm{pp}}^{\mathrm{e}})^2k^2C_0^4(k)}$$
$$\times\Big(E_{\mathrm{e}^-}E_{\nu_{\mathrm{e}}}-\frac{1}{3}\mathbf{k}_{\mathrm{e}^-}\cdot\mathbf{k}_{\nu_{\mathrm{e}}}\Big). \tag{8.2}$$

where $F_+(Z,E_{\mathrm{e}^-})$ is the Fermi function [45] describing the Coulomb interaction of the electron with the nuclear system having a charge Z. In the case of the reaction $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$ we have $Z = 2$. At $\alpha^2Z^2 \ll 1$ the Fermi function $F_+(Z,E_{\mathrm{e}^-})$ reads [45]

$$F_+(Z,E_{\mathrm{e}^-}) = \frac{2\pi\eta_{\mathrm{e}^-}}{1-e^{-2\pi\eta_{\mathrm{e}^-}}}, \tag{8.3}$$

where $\eta_{\mathrm{e}^-} = Z\alpha/v_{\mathrm{e}^-} = Z\alpha E_{\mathrm{e}^-}/\sqrt{E_{\mathrm{e}^-}^2-m_{\mathrm{e}^-}^2}$ and v_{e^-} is a velocity of the electron.

For the evaluation of the r.h.s. of eq. (8.2) we have also used the expression for the astrophysical factor

$$S_{\mathrm{pp}}(0) = \frac{9g_{\mathrm{A}}^2G_{\mathrm{V}}^2Q_{\mathrm{D}}M_{\mathrm{N}}^3}{1280\pi^4r_{\mathrm{C}}}\,C_{\mathrm{NN}}^2\,m_{\mathrm{e}}^5\,\Omega_{\mathrm{De}^+\nu_{\mathrm{e}}}, \tag{8.4}$$

where $\Omega_{\mathrm{De}^+\nu_{\mathrm{e}}} = (W/m_{\mathrm{e}})^5f(m_{\mathrm{e}}/W) = 4.481$ at $W = 0.932\,\mathrm{MeV}$. The function $f(m_{\mathrm{e}}/W)$ is defined by eq. (7.4).

In the rest frame of the deuteron the cross-section for the reaction $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$ is defined by

$$\sigma^{\nu_{\mathrm{e}}\mathrm{D}\to\mathrm{e}^-\mathrm{pp}}(E_{\nu_{\mathrm{e}}}) =$$
$$\frac{1}{4M_{\mathrm{D}}E_{\nu_{\mathrm{e}}}}\int\overline{|\mathcal{M}(\nu_{\mathrm{e}}+\mathrm{D}\to\mathrm{e}^-+\mathrm{p}+\mathrm{p})|^2}$$
$$\times\frac{1}{2}\,(2\pi)^4\,\delta^{(4)}(k_{\mathrm{D}}+k_{\nu_{\mathrm{e}}}-p_1-p_2-k_{\mathrm{e}^-})$$
$$\times\frac{\mathrm{d}^3p_1}{(2\pi)^32E_1}\,\frac{\mathrm{d}^3p_2}{(2\pi)^32E_2}\,\frac{\mathrm{d}^3k_{\mathrm{e}^-}}{(2\pi)^32E_{\mathrm{e}^-}}, \tag{8.5}$$

where $E_{\nu_{\mathrm{e}}}$, E_1, E_2 and E_{e^-} are the energies of the neutrino, the protons and the electron. The integration over the phase volume of the (ppe^-) state we perform in the non-relativistic limit and in the rest frame of the deuteron,

see equation (8.6) *on next page*

where T_{e^-} is the kinetic energy of the electron, E_{th} is the neutrino energy threshold: $E_{\mathrm{th}} = \varepsilon_{\mathrm{D}} + m_{\mathrm{e}} - (M_{\mathrm{n}} - M_{\mathrm{p}}) = (2.225 + 0.511 - 1.293)\,\mathrm{MeV} = 1.443\,\mathrm{MeV}$. The function $\Omega_{\mathrm{ppe}^-}(y)$, where $y = E_{\nu_{\mathrm{e}}}/E_{\mathrm{th}}$ and $E_{\mathrm{th}} = \varepsilon_{\mathrm{D}} - (M_{\mathrm{n}} -$

[3] See the last paragraph of sect. 3 and the first paragraph of sect. 5 of ref. [31].

$$\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3k_e}{(2\pi)^3 2E_{e^-}}(2\pi)^4\,\delta^{(4)}(k_D+k_{\nu_e}-p_1-p_2-k_{e^-})$$
$$\times\frac{C_0^2(\sqrt{M_N T_{pp}})\,F_D^2(M_N T_{pp})\,F_+(Z,E_{e^-})}{\Big[1-\frac{1}{2}a^e_{pp}r^e_{pp}M_N T_{pp}+\frac{a^e_{pp}}{r_C}\,h(2r_C\sqrt{M_N T_{pp}})\Big]^2+(a^e_{pp})^2 M_N T_{pp}C_0^4(\sqrt{M_N T_{pp}})}\Big(E_{e^-}E_{\nu_e}-\frac{1}{3}\mathbf{k}_{e^-}\cdot\mathbf{k}_{\nu_e}\Big)=$$
$$\frac{E_{\nu_e}M_N^3}{128\pi^3}\left(\frac{E_{th}}{M_N}\right)^{7/2}\left(\frac{2m_e}{E_{th}}\right)^{3/2}\frac{1}{E_{th}^2}\iint dT_{e^-}dT_{pp}\delta(E_{\nu_e}-E_{th}-T_{e^-}-T_{pp})\sqrt{T_{e^-}T_{pp}}\left(1+\frac{T_{e^-}}{m_e}\right)\sqrt{1+\frac{T_{e^-}}{2m_e}}$$
$$\times\frac{C_0^2(\sqrt{M_N T_{pp}})\,F_D^2(M_N T_{pp})\,F_+(Z,E_{e^-})}{\Big[1-\frac{1}{2}a^e_{pp}r^e_{pp}M_N T_{pp}+\frac{a^e_{pp}}{r_C}\,h(2r_C\sqrt{M_N T_{pp}})\Big]^2+(a^e_{pp})^2 M_N T_{pp}C_0^4(\sqrt{M_N T_{pp}})}=\frac{E_{\nu_e}M_N^3}{128\pi^3}\left(\frac{E_{th}}{M_N}\right)^{7/2}\left(\frac{2m_e}{E_{th}}\right)^{3/2}(y-1)^2\,\Omega_{ppe^-}(y),\qquad(8.6)$$

$M_p)+m_e=1.443\,\mathrm{MeV}$, is determined by the integral

$$\Omega_{ppe^-}(y)=\int_0^1 dx\sqrt{x(1-x)}\Big(1+\frac{E_{th}}{m_e}(y-1)(1-x)\Big)$$
$$\times\sqrt{1+\frac{E_{th}}{2m_e}(y-1)(1-x)}C_0^2(\sqrt{M_N E_{th}\,(y-1)\,x})$$
$$\times F_D^2(M_N E_{th}\,(y-1)\,x)\,F_+(Z,m_e+E_{th}(y-1)\,(1-x))$$
$$\times\Bigg\{\Bigg[1-\frac{1}{2}a^e_{pp}r^e_{pp}M_N E_{th}\,(y-1)\,x$$
$$+\frac{a^e_{pp}}{r_C}\,h(2r_C\sqrt{M_N E_{th}\,(y-1)\,x})\Bigg]^2$$
$$+(a^e_{pp})^2\,M_N E_{th}\,(y-1)\,xC_0^4$$
$$\times(\sqrt{M_N E_{th}\,(y-1)\,x})\Bigg\}^{-1},\qquad(8.7)$$

where we have changed the variable $T_{pp}=(E_{\nu_e}-E_{th})\,x$.

The cross-section for the reaction $\nu_e+D\to e^-+p+p$ is defined by

$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e})=$$
$$S_{pp}(0)\,\frac{640r_C}{\pi\Omega_{D^+\nu_e}}\left(\frac{M_N}{E_{th}}\right)^{3/2}\left(\frac{E_{th}}{2m_e}\right)^{7/2}(y-1)^2\,\Omega_{ppe^-}(y)$$
$$=3.69\times10^5\,S_{pp}(0)\,(y-1)^2\Omega_{ppe^-}(y),\qquad(8.8)$$

where $S_{pp}(0)$ is measured in $\mathrm{MeV\,cm^2}$. For $S_{pp}(0)=4.08\times10^{-49}\,\mathrm{MeV\,cm^2}$ eq. (7.7) the cross-section $\sigma^{\nu_e D\to e^- pp}(E_{\nu_e})$ reads

$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e})=1.50\,(y-1)^2\,\Omega_{ppe^-}(y)\,10^{-43}\,\mathrm{cm^2}.\qquad(8.9)$$

Recently the calculation of the cross-section for the reaction $\nu_e+D\to e^-+p+p$ has been carried in ref. [46] within the PMA and tabulated for the neutrino energies ranging over the region from threshold up to 170 MeV. Since our result is valid for much lower neutrino energies, we give the numerical values of the cross-section only for the energies from the region $4\,\mathrm{MeV}\le E_{\nu_e}\le 10\,\mathrm{MeV}$:

$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e}=4\,\mathrm{MeV})=2.46\,(1.577)\times10^{-43}\,\mathrm{cm^2},$$
$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e}=6\,\mathrm{MeV})=10.04\,(6.239)\times10^{-43}\,\mathrm{cm^2},$$
$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e}=8\,\mathrm{MeV})=2.37\,(1.463)\times10^{-42}\,\mathrm{cm^2},$$
$$\sigma^{\nu_e D\to e^- pp}(E_{\nu_e}=10\,\mathrm{MeV})=4.39\,(2.708)\times10^{-43}\,\mathrm{cm^2}.\qquad(8.10)$$

The data in parentheses are taken from Table 1 of ref. [46]. Thus, on the average the cross-section $\sigma^{\nu_e D\to e^- pp}(E_{\nu_e})$ calculated in the NNJL model by a factor of 1.6 is larger compared with the PMA ones.

Since the amplitude of the reaction $\nu_e+D\to e^-+p+p$ is completely defined by the amplitude of the solar proton burning $p+p\to D+e^++\nu_e$ that is described in the NNJL model in agreement with other theoretical approaches, our prediction for the cross-section for the neutrino disintegration of the deuteron eq. (8.10) is a challenge to the experiments planned at SNO [35].

In order to compare the cross-section for the neutrino disintegration of the deuteron $\nu_e+D\to e^-+p+p$ with experimental data planned to be obtained by SNO Collaboration we should average it over the ^8B solar neutrino energy spectrum produced by the β decay $^8B\to{}^8Be^*+e^++\nu_e$ in the solar core. Using the ^8B solar neutrino energy spectrum [47] and integrating over the region $5\,\mathrm{MeV}\le E_{\nu_e}\le 15\,\mathrm{MeV}$ [48], where the lower bound is related to the experimental threshold of experiments at SNO and the upper one is defined by the kinematics of the decay $^8B\to{}^8Be^*+e^++\nu_e$, we get

$$\langle\sigma^{\nu_e D\to e^- pp}(E_{\nu_e})\rangle_{\Phi(^8B)}=2.62\times10^{-42}\,\mathrm{cm^2}.\qquad(8.11)$$

For the comparison of the theoretical cross-section with the experimental one measured at SNO one should take into account a possible decrease of the experimental value in the case of the existence of neutrino flavour oscillations [27,48].

9 The astrophysical factor for the pep process

In the NNJL model the amplitude of the reaction p + e^- + p → D + ν_e or the pep-process is related to the amplitude of the solar proton burning p + p → D + e^+ + ν_e eq. (6.17) as well as to the amplitude of the neutrino disintegration of the deuteron ν_e + D → e^- + p + p. Indeed, the effective Lagrangians $\mathcal{L}_{\rm eff}^{{\rm pe^-p}\to{\rm D}\nu_{\rm e}}(x)$, $\mathcal{L}_{\rm eff}^{{\rm pp}\to{\rm De^+}\nu_{\rm e}}(x)$ and $\mathcal{L}_{\rm eff}^{\nu_{\rm e}{\rm D}\to{\rm e^-pp}}(x)$ are defined by the anomaly of the one-nucleon loop triangle AAV-diagrams with two axial-vector (A) and one vector (V) vertices (see appendix). The amplitude of the pep-process reads

$$\begin{aligned}&i\mathcal{M}({\rm p}+{\rm e}^-+{\rm p}\to{\rm D}+\nu_e)=g_{\rm A}M_{\rm N}\,C_{\rm NN}\,G_{\rm V}\,\frac{3g_{\rm V}}{4\pi^2}\\&\times e^*_\mu(k_{\rm D})\,[\bar u(k_{\nu_{\rm e}})\gamma^\mu(1-\gamma^5)u(k_{\rm e^-})]\\&\times\frac{C_0(k)}{1-\dfrac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2+\dfrac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2kr_{\rm C})+i\,a^{\rm e}_{\rm pp}\,k\,C_0^2(k)}\\&\times[\bar{u^c}(p_2)\gamma^5u(p_1)]\,F_{\rm D}(k^2).\end{aligned}\qquad(9.1)$$

The amplitude eq. (9.1) squared, averaged and summed over polarizations of the interacting particles is defined by

$$\begin{aligned}&\overline{|\mathcal{M}({\rm p}+{\rm e}^-+{\rm p}\to{\rm D}+\nu_e)|^2}=\\&g^2_{\rm A}M^6_{\rm N}\,C^2_{\rm NN}\,G^2_{\rm V}\,\frac{27Q_{\rm D}}{\pi^2}\,F^2_{\rm D}(k^2)\,F_+(Z,E_{\rm e^-})\\&\times\frac{C_0^2(k)}{\Big[1-\dfrac{1}{2}\,a^{\rm e}_{\rm pp}r^{\rm e}_{\rm pp}k^2+\dfrac{a^{\rm e}_{\rm pp}}{r_{\rm C}}\,h(2kr_{\rm C})\Big]^2+(a^{\rm e}_{\rm pp})^2k^2C_0^4(k)}\\&\times\Big(E_{\rm e^+}E_{\nu_{\rm e}}-\frac{1}{3}\mathbf{k}_{\rm e^+}\cdot\mathbf{k}_{\nu_{\rm e}}\Big),\end{aligned}\qquad(9.2)$$

where $F_+(Z,E_{\rm e^-})$ is the Fermi function given by eq. (8.3).

At low energies the cross-section $\sigma^{{\rm pe^-p}\to{\rm D}\nu_{\rm e}}(T_{\rm pp})$ for the pep-process can be determined as follows [49]

$$\begin{aligned}&\sigma^{{\rm pe^-p}\to{\rm D}\nu_{\rm e}}(T_{\rm pp})=\frac{1}{v}\frac{1}{4M^2_{\rm N}}\int\frac{{\rm d}^3k_{\rm e^-}}{(2\pi)^32E_{\rm e^-}}\,g\,n(\mathbf{k}_{\rm e^-})\\&\times\int\overline{|\mathcal{M}({\rm p}+{\rm e}^-+{\rm p}\to{\rm D}+\nu_{\rm e})|^2}\\&\times(2\pi)^4\delta^{(4)}(k_{\rm D}+k_{\nu_{\rm e}}-p_1-p_2-k_{\rm e^-})\frac{{\rm d}^3k_{\rm D}}{(2\pi)^32M_{\rm D}}\frac{{\rm d}^3k_{\nu_{\rm e}}}{(2\pi)^32E_{\nu_{\rm e}}},\end{aligned}\qquad(9.3)$$

where $g=2$ is the number of the electron spin states and v is a relative velocity of the pp pair. The electron distribution function $n(\mathbf{k}_{\rm e^-})$ can be taken in the form [45]

$$n(\mathbf{k}_{\rm e^-})=e^{\bar\nu\,-\,T_{\rm e^-}/kT_{\rm c}},\qquad(9.4)$$

where $k=8.617\times10^{-11}\,{\rm MeV\,K^{-1}}$, $T_{\rm c}$ is a temperature of the core of the Sun. The distribution function $n(\mathbf{k}_{\rm e^-})$ is normalized by the condition

$$g\int\frac{{\rm d}^3k_{\rm e^-}}{(2\pi)^3}\,n(\mathbf{k}_{\rm e^-})=n_{\rm e^-},\qquad(9.5)$$

where $n_{\rm e^-}$ is the electron number density. From the normalization condition eq. (9.5) we derive

$$e^{\bar\nu}=\frac{4\,\pi^3\,n_{\rm e^-}}{(2\pi\,m_{\rm e}\,kT_{\rm c})^{3/2}}.\qquad(9.6)$$

The astrophysical factor $S_{\rm pep}(0)$ is then defined by

$$\begin{aligned}&S_{\rm pep}(0)=S_{\rm pp}(0)\,\frac{15}{2\pi}\,\frac{1}{\Omega_{\rm De^+\nu_e}}\,\frac{1}{m^3_{\rm e}}\left(\frac{E_{\rm th}}{m_{\rm e}}\right)^2\\&\times e^{\bar\nu}\int{\rm d}^3k_{\rm e^-}\,e^{-T_{\rm e^-}/kT_{\rm c}}\,F_+(Z,E_{\rm e^-}).\end{aligned}\qquad(9.7)$$

For the ratio $S_{\rm pep}(0)/S_{\rm pp}(0)$ we obtain

$$\begin{aligned}&\frac{S_{\rm pep}(0)}{S_{\rm pp}(0)}=\frac{2^{3/2}\pi^{5/2}}{f_{\rm pp}(0)}\left(\frac{\alpha Zn_{\rm e^-}}{m^3_{\rm e}}\right)\\&\times\left(\frac{E_{\rm th}}{m_{\rm e}}\right)^2\sqrt{\frac{m_{\rm e}}{kT_{\rm c}}}\,I\left(Z\sqrt{\frac{2m_{\rm e}}{kT_{\rm c}}}\right).\end{aligned}\qquad(9.8)$$

We have set $f_{\rm pp}(0)=\Omega_{\rm De^+\nu_e}/30=0.149$ [45] and the function $I(x)$ introduced by Bahcall and May [45] reads

$$I(x)=\int\limits_0^\infty\frac{{\rm d}u\,e^{-u}}{1-e^{-\pi\alpha\,x/\sqrt{u}}}.\qquad(9.9)$$

The relation between the astrophysical factors $S_{\rm pep}(0)$ and $S_{\rm pp}(0)$ given by eq. (9.8) is in complete agreement with that obtained by Bahcall and May [45].

By virtue of the direct relation between the amplitudes of the pep-process and the reaction for the neutrino disintegration of the deuteron ν_e + D → e^- + p + p that we have in the NNJL model the agreement with the result obtained by Bahcall and May [45] is on favour of our predictions for the cross-section for the reaction ν_e + D → e^- + p + p.

10 The reaction $\bar\nu_e$ + D → e^+ + n + n

The effective Lagrangian $\mathcal{L}_{\rm eff}^{\bar\nu_{\rm e}{\rm D}\to{\rm e^+nn}}(x)$ of the low-energy nuclear transition $\bar\nu_e$ + D → e^+ + n + n we evaluate by analogy with $\mathcal{L}_{\rm eff}^{{\rm pp}\to{\rm De^+}\bar\nu_{\rm e}}(x)$ (see eq. (A.27)) of the appendix) through the one-nucleon loop exchanges and at leading order in the large $N_{\rm C}$ expansion. The effective Lagrangian $\mathcal{L}_{\rm eff}^{\bar\nu_{\rm e}{\rm D}\to{\rm e^+nn}}(x)$ is defined by the anomaly of the one-nucleon loop triangle AAV-diagram as well as the effective Lagrangian $\mathcal{L}_{\rm eff}^{{\rm pp}\to{\rm De^+}\nu_{\rm e}}(x)$. The result reads

$$\begin{aligned}&\mathcal{L}_{\rm eff}^{\bar\nu_{\rm e}{\rm D}\to{\rm e^+nn}}(x)=ig_{\rm A}M_{\rm N}C_{\rm NN}\frac{G_{\rm V}}{\sqrt2}\frac{3g_{\rm V}}{4\pi^2}\\&\times D_\mu(x)\,[\bar n(x)\gamma^5n^c(x)]\,[\bar\psi_{\nu_{\rm e}}(x)\gamma^\mu(1-\gamma^5)\psi_{\rm e}(x)].\end{aligned}\qquad(10.1)$$

The amplitude of the reaction $\bar{\nu}_e + D \to e^+ + n + n$ we obtain in the form

$$i\mathcal{M}(\bar{\nu}_e + D \to e^+ + n + n) = - g_A M_N C_{NN} \frac{G_V}{\sqrt{2}} \frac{3g_V}{2\pi^2} \frac{1}{1 - \frac{1}{2} r_{nn} a_{nn} k^2 + i\, a_{nn}\, k} F_D(k^2) \times e_\mu(k_D) [\bar{v}(k_{\bar{\nu}_e})\gamma^\mu(1-\gamma^5)v(k_{e^+})] [\bar{u}(p_2)\gamma^5 u^c(p_1)], \quad (10.2)$$

where the form factor $F_D(k^2)$ describes a spatial smearing of the deuteron eq. (2.16). The factor $1/(1 - \frac{1}{2}r_{nn}a_{nn}k^2 + i\, a_{nn}\, k)$ gives the contribution of low-energy nuclear forces to the relative movement of the nn pair in the ${}^1S_0$ state. The result is obtained by the summation of an infinite series of the one-neutron bubbles evaluated at leading order in the large N_C expansion. Since we work in the isotopical limit, we set $a_{nn} = a_{np} = -23.75$ fm and $r_{nn} = r_{np} = 2.75$ fm. The recent experimental values of the S wave scattering length and the effective range of low-energy elastic nn scattering are equal to $a_{nn} = (-18.8 \pm 0.3)$ fm and $r_{nn} = (2.75 \pm 0.11)$ fm [50,51].

The amplitude eq. (10.2), squared, averaged over polarizations of the deuteron and summed over polarizations of the final particles, reads

$$\overline{|\mathcal{M}(\bar{\nu}_e + D \to e^+ + n + n)|^2} = \frac{144}{\pi^2} \frac{Q_D g_A^2 G_V^2 C_{NN}^2 M_N^6 F_D^2(k^2)}{\left(1 - \frac{1}{2} r_{nn} a_{nn} k^2\right)^2 + a_{nn}^2 k^2} \times \left(E_{e^+} E_{\bar{\nu}_e} - \frac{1}{3}\mathbf{k}_{e^+}\cdot \mathbf{k}_{\bar{\nu}_e}\right). \quad (10.3)$$

The cross-section for the reaction $\bar{\nu}_e + D \to e^+ + n + n$ is defined by

$$\sigma^{\bar{\nu}_e D \to e^+ nn}(E_{\bar{\nu}_e}) = \frac{1}{4E_D E_{\bar{\nu}_e}} \int \overline{|\mathcal{M}(\bar{\nu}_e + D \to e^+ + n + n)|^2} \times \frac{1}{2}(2\pi)^4 \delta^{(4)}(Q + k_{\bar{\nu}_e} - p_1 - p_2 - k_{e^+}) \times \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3k_{e^+}}{(2\pi)^3 2E_{e^+}}, \quad (10.4)$$

where E_D, $E_{\bar{\nu}_e}$, E_1, E_2 and E_{e^+} are the energies of the deuteron, the antineutrino, the neutrons and the positron. The integration over the phase volume of the (nne$^+$) state we perform in the non-relativistic limit and in the rest frame of the deuteron

$$\frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3k_{e^+}}{(2\pi)^3 2E_{e^+}} (2\pi)^4 \times \delta^{(4)}(Q + k_{\bar{\nu}_e} - p_1 - p_2 - k_{e^+}) \times \frac{\left(E_{e^+}E_{\bar{\nu}_e} - \frac{1}{3}\mathbf{k}_{e^+}\cdot\mathbf{k}_{\bar{\nu}_e}\right) F_D^2(M_N T_{nn})}{\left(1 - \frac{1}{2} r_{nn} a_{nn} M_N T_{nn}\right)^2 + a_{nn}^2 M_N T_{nn}} =$$

$$\frac{E_{\bar{\nu}_e} M_N^3}{1024\pi^2} \left(\frac{E_{th}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{th}}\right)^{3/2} \frac{8}{\pi E_{th}^2} \times \iint dT_{e^+} dT_{nn} \frac{\sqrt{T_{e^+} T_{nn}}\, F_D^2(M_N T_{nn})}{\left(1 - \frac{1}{2} r_{nn} a_{nn} M_N T_{nn}\right)^2 + a_{nn}^2 M_N T_{nn}} \times \left(1 + \frac{T_{e^+}}{m_e}\right) \sqrt{1 + \frac{T_{e^+}}{2m_e}}\, \delta\left(E_{\bar{\nu}_e} - E_{th} - T_{e^+} - T_{nn}\right) =$$

$$= \frac{E_{\bar{\nu}_e} M_N^3}{1024\pi^2} \left(\frac{E_{th}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{th}}\right)^{3/2} (y-1)^2\, \Omega_{nn\bar{\nu}_e}(y), \quad (10.5)$$

where T_{nn} is the kinetic energy of the nn pair in the ${}^1S_0$ state, T_{e^+} and $m_e = 0.511$ MeV are the kinetic energy and the mass of the positron, $y = E_{\bar{\nu}_e}/E_{th}$ and E_{th} is the antineutrino energy threshold of the reaction $\bar{\nu}_e + D \to e^+ + n + n$: $E_{th} = \varepsilon_D + m_e + (M_n - M_p) = (2.225 + 0.511 + 1.293)$ MeV $= 4.029$ MeV. The function $\Omega_{nn\bar{\nu}_e}(y)$ is defined by

$$\Omega_{nn\bar{\nu}_e}(y) = \frac{8}{\pi}\int_0^1 dx \frac{\sqrt{x(1-x)}\, F_D^2(M_N E_{th}(y-1)x)}{\left(1 - \frac{1}{2} r_{nn} a_{nn} M_N E_{th}(y-1)x\right)^2 + a_{nn}^2 M_N E_{th}(y-1)x} \times \left(1 + \frac{E_{th}}{m_e}(y-1)(1-x)\right)\sqrt{1 + \frac{E_{th}}{2m_e}(y-1)(1-x)}, \quad (10.6)$$

where we have changed the variable $T_{nn} = (E_{\bar{\nu}_e} - E_{th})\,x$. The function $f(y)$ is normalized to unity at $y = 1$, *i.e.* at threshold $E_{\bar{\nu}_e} = E_{th}$. Thus, the cross-section for the reaction $\bar{\nu}_e + D \to e^+ + n + n$ reads

$$\sigma^{\bar{\nu}_e D \to e^+ nn}(E_{\bar{\nu}_e}) = \sigma_0\, (y-1)^2\, \Omega_{nn\bar{\nu}_e}(y), \quad (10.7)$$

where σ_0 amounts to

$$\sigma_0 = Q_D\, C_{NN}^2 \frac{9 g_A^2 G_V^2 M_N^8}{512\pi^4} \left(\frac{E_{th}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{th}}\right)^{3/2} = 4.58 \times 10^{-43}\, \mathrm{cm}^2. \quad (10.8)$$

The experimental data on the antineutrino disintegration of the deuteron are given in terms of the cross-section averaged over the antineutrino energy spectrum [36]:

$$\langle \sigma^{\bar{\nu}_e D \to e^+ nn}(E_{\bar{\nu}_e})\rangle_{exp} = (9.83 \pm 2.04) \times 10^{-45}\, \mathrm{cm}^2. \quad (10.9)$$

In order to average the theoretical cross-section eq. (10.7) over the antineutrino spectrum we should use the spectrum given by Table YII of ref. [36]. This yields

$$\langle\sigma^{\bar{\nu}_{\rm e}{\rm D}\to {\rm e}^+{\rm nn}}(E_{\bar{\nu}_{\rm e}})\rangle = 11.56\times 10^{-45}\,{\rm cm}^2. \qquad (10.10)$$

The theoretical value eq. (10.10)[4] agrees well with the experimental one eq. (10.9).

11 The reactions $\nu_{\rm e}(\bar{\nu}_{\rm e})$ + D → $\nu_{\rm e}(\bar{\nu}_{\rm e})$ + n + p

The amplitude of the neutrino disintegration of the deuteron caused by neutral weak current $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p evaluated through one-nucleon loop exchanges and at leading order in the large $N_{\rm C}$ expansion reads (see eq. (A.30) of the appendix):

$$\begin{aligned} &i\mathcal{M}(\nu_{\rm e}+{\rm D}\to\nu_{\rm e}+{\rm n}+{\rm p}) = \\ &-g_{\rm A}\,M_{\rm N}\,C_{\rm NN}\,\frac{G_{\rm F}}{\sqrt{2}}\,\frac{3g_{\rm V}}{4\pi^2}\,\frac{1}{1-\frac{1}{2}r_{\rm np}a_{\rm np}k^2+i\,a_{\rm np}\,k}\,F_{\rm D}(k^2)\\ &\times e_{\mu}(k_{\rm D})\,[\bar{u}(k'_{\nu_{\rm e}})\gamma^{\mu}(1-\gamma^5)u(k_{\nu_{\rm e}})][\bar{u}(p_2)\gamma^5u^c(p_1)]. \end{aligned} \qquad (11.1)$$

The amplitude eq. (11.1) squared, averaged over polarizations of the deuteron, summed over polarizations of the nucleons reads

$$\begin{aligned} &\overline{|\mathcal{M}(\nu_{\rm e}+{\rm D}\to\nu_{\rm e}+{\rm n}+{\rm p})|^2} = \\ &\frac{36}{\pi^2}\,\frac{Q_{\rm D}g_{\rm A}^2G_{\rm F}^2C_{\rm NN}^2M_{\rm N}^6F_{\rm D}^2(k^2)}{\Big(1-\frac{1}{2}r_{\rm np}a_{\rm np}k^2\Big)^2+a_{\rm np}^2k^2}\Big(E'_{\nu_{\rm e}}E_{\nu_{\rm e}}-\frac{1}{3}{\bf k}'_{\nu_{\rm e}}{\bf \cdot k}_{\nu_{\rm e}}\Big). \end{aligned} \qquad (11.2)$$

In the rest frame of the deuteron the cross-section for the reaction $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p is defined by

$$\begin{aligned} &\sigma^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(E_{\bar{\nu}_{\rm e}}) = \frac{1}{4M_{\rm D}E_{\nu_{\rm e}}}\\ &\times\int\overline{|\mathcal{M}(\nu_{\rm e}+{\rm D}\to\nu_{\rm e}+{\rm n}+{\rm p})|^2}\\ &\times(2\pi)^4\,\delta^{(4)}(k_{\rm D}+k_{\nu_{\rm e}}-p_1-p_2-k'_{\nu_{\rm e}})\\ &\times\frac{{\rm d}^3p_1}{(2\pi)^32E_1}\frac{{\rm d}^3p_2}{(2\pi)^32E_2}\frac{{\rm d}^3k'_{\nu_{\rm e}}}{(2\pi)^32E'_{\nu_{\rm e}}}. \end{aligned} \qquad (11.3)$$

The integration over the phase volume of the (np$\nu_{\rm e}$) state we perform in the non-relativistic limit and in the rest frame of the deuteron,

$$\begin{aligned} &\int\frac{{\rm d}^3p_1}{(2\pi)^32E_1}\frac{{\rm d}^3p_2}{(2\pi)^32E_2}\frac{{\rm d}^3k'_{\nu_{\rm e}}}{(2\pi)^32E'_{\nu_{\rm e}}}\\ &\times(2\pi)^4\,\delta^{(4)}(k_{\rm D}+k_{\nu_{\rm e}}-p_1-p_2-k'_{\nu_{\rm e}})\Big(E_{\nu_{\rm e}}E'_{\nu_{\rm e}}-\frac{1}{3}{\bf k}_{\nu_{\rm e}}\cdot{\bf k}'_{\nu_{\rm e}}\Big)\\ &\times\frac{F_{\rm D}^2(M_{\rm N}T_{\rm np})}{\Big(1-\frac{1}{2}r_{\rm np}a_{\rm np}M_{\rm N}T_{\rm np}\Big)^2+a_{\rm np}^2M_{\rm N}T_{\rm np}} = \\ &\frac{E_{\nu_{\rm e}}M_{\rm N}^3}{210\pi^3}\left(\frac{E_{\rm th}}{M_{\rm N}}\right)^{7/2}(y-1)^{7/2}\,\Omega_{{\rm np}\nu_{\rm e}}(y). \end{aligned} \qquad (11.4)$$

The function $\Omega_{{\rm np}\nu_{\rm e}}(y)$, where $y = E_{\bar{\nu}_{\rm e}}/E_{\rm th}$ and $E_{\rm th} = \varepsilon_{\rm D} = 2.225\,{\rm MeV}$ is threshold of the reaction, is defined by the integral

$$\begin{aligned} &\Omega_{{\rm np}\nu_{\rm e}}(y) = \\ &\frac{105}{16}\int_0^1{\rm d}x\frac{\sqrt{x}\,(1-x)^2}{\Big(1-\frac{1}{2}\frac{r_{\rm np}a_{\rm np}}{r_{\rm D}^2}(y-1)x\Big)^2+\frac{a_{\rm np}^2}{r_{\rm D}^2}(y-1)\,x}\\ &\times\frac{1}{(1+(y-1)\,x)^2}, \end{aligned} \qquad (11.5)$$

where we have changed the variable $T_{\rm np} = (E_{\bar{\nu}_{\rm e}} - E_{\rm th})\,x$ and used the relation $M_{\rm N}E_{\rm th} = 1/r_{\rm D}^2$ at $E_{\rm th} = \varepsilon_{\rm D}$. The function $\Omega_{{\rm np}\nu_{\rm e}}(y)$ is normalized to unity at $y = 1$, *i.e.* at threshold $E_{\bar{\nu}_{\rm e}} = E_{\rm th}$.

The cross-section for the reaction $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p reads

$$\sigma^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(E_{\nu_{\rm e}}) = \sigma_0\,(y-1)^{7/2}\,\Omega_{{\rm np}\nu_{\rm e}}(y), \qquad (11.6)$$

where σ_0 amounts to

$$\begin{aligned} \sigma_0 &= Q_{\rm D}\,C_{\rm NN}^2\,\frac{3g_{\rm A}^2\,G_{\rm F}^2M_{\rm N}^8}{140\pi^5}\left(\frac{E_{\rm th}}{M_{\rm N}}\right)^{7/2}\\ &= 1.84\times10^{-43}\,{\rm cm}^2. \end{aligned} \qquad (11.7)$$

In our approach the cross-section for the disintegration of the deuteron by neutrinos $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p coincides with the cross-section for the disintegration of the deuteron by antineutrinos $\bar{\nu}_{\rm e}$ + D → $\bar{\nu}_{\rm e}$ + n + p, $\sigma^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(E_{\nu_{\rm e}}) = \sigma^{\bar{\nu}_{\rm e}{\rm D}\to\bar{\nu}_{\rm e}{\rm np}}(E_{\bar{\nu}_{\rm e}})$. Therefore, we have an opportunity to compare our result with the experimental data on the disintegration of the deuteron by antineutrinos [36]. The experimental value of the cross-section for the antineutrino disintegration of the deuteron $\bar{\nu}_{\rm e}$ + D → $\bar{\nu}_{\rm e}$ + n + p averaged over the antineutrino spectrum reads [36]

$$\langle\sigma^{\bar{\nu}_{\rm e}{\rm D}\to\bar{\nu}_{\rm e}{\rm np}}(E_{\bar{\nu}_{\rm e}})\rangle_{\rm exp} = (6.08\pm0.77)\times10^{-45}\,{\rm cm}^2. \qquad (11.8)$$

By using the antineutrino spectrum given by Table YII of ref. [36] for the calculation of the average value of the theoretical cross-section eq. (11.6) we obtain

$$\langle\sigma^{\bar{\nu}_{\rm e}{\rm D}\to\bar{\nu}_{\rm e}{\rm np}}(E_{\bar{\nu}_{\rm e}})\rangle = 6.28\times10^{-45}\,{\rm cm}^2. \qquad (11.9)$$

[4] This result is obtained at zero contribution of the nucleon tensor current (see appendix and the discussion in the conclusion).

The theoretical value eq. (11.9)[5] agrees well with the experimental one eq. (11.8).

The cross-section for the neutrino disintegration of the deuteron $\nu_e + D \to \nu_e + n + p$ averaged over the ^{8}B solar neutrino spectrum [47] for energy region $5\,\mathrm{MeV} \le E_{\nu_e} \le 15\,\mathrm{MeV}$ is given by

$$\langle \sigma^{\nu_e D \to \nu_e np}(E_{\nu_e}) \rangle_{\Phi(^8B)} = 1.85 \times 10^{-43}\,\mathrm{cm}^2. \qquad (11.10)$$

This result can be directly compared with the experimental data obtained by SNO, since the averaged value for the cross-section for the reaction $\nu_e + D \to \nu_e + n + p$ caused by the neutral weak current should not depend on whether neutrino flavours oscillate or not [27,48], of course if no *sterile* neutrinos exist.

12 Conclusion

We have considered the description of a dynamics of low-energy nuclear forces within the Nambu-Jona-Lasinio model of light nuclei (the NNJL model) for low-energy electromagnetic and weak nuclear reactions with the deuteron. We have shown that the NNJL model enables to describe in a reasonable agreement with both experimental data and other theoretical approaches all variety of low-energy electromagnetic and weak nuclear reactions with the deuteron in the final or initial state coupled to a nucleon-nucleon (NN) pair in the $^1S_0$ state. In the bulk the reaction rates for the neutron-proton radiative capture $n + p \to D + \gamma$ for thermal neutrons, the photomagnetic disintegration of the deuteron $\gamma + D \to n + p$, the solar proton burning $p + p \to D + e^+ + \nu_e$, the pep-process $p + e^- + p \to D + \nu_e$, the disintegration of the deuteron by neutrinos and antineutrinos caused by charged $\nu_e + D \to e^- + p + p$ and $\bar{\nu}_e + D \to e^+ + n + n$ and neutral $\nu_e(\bar{\nu}_e) + D \to \nu_e(\bar{\nu}_e) + n + p$ weak currents are calculated in agreement with other theorical approaches and experimental data.

When matching our results with those obtained in the PMA we would like to emphasize a much more simple description of the NN interaction responsible for the transition $N + N \to N + N$ and a substantial simplification of the evaluation of matrix elements of low-energy nuclear transitions near thresholds of the reactions where the NN pair in the $^1S_0$ state is produced or absorbed with a relative momentum p comparable with zero. Such a simplification is rather clear in the NNJL model where with a good accuracy the deuteron can be considered within a quantum field-theoretic approach as a point-like particle. Indeed, the spatial region of the localization of the NN pair is of order of $O(1/p)$. Near thresholds the effective radius of the deuteron $r_D = 4.319\,\mathrm{MeV}$ is much smaller than $1/p$, $r_D \ll 1/p$. This yields that the NN pair does not feel the spatial smearing of the deuteron and couples to the deuteron as to a point-like particle. A correct description of strong interactions of the point-like deuteron coupled to the NN pair in the $^1S_0$ state is guaranteed then by the one-nucleon loop exchanges with dominant contributions of the nucleon-loop anomalies. This implies that the effective Lagrangians $\mathcal{L}_{\mathrm{eff}}^{np\to D\gamma}(x)$, $\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x)$, $\mathcal{L}_{\mathrm{eff}}^{\nu_e D\to e^- pp}(x)$, $\mathcal{L}_{\mathrm{eff}}^{pe^- p\to D\nu_e}(x)$, $\mathcal{L}_{\mathrm{eff}}^{\bar{\nu}_e D\to e^+ nn}(x)$ and $\mathcal{L}_{\mathrm{eff}}^{\bar{\nu}_e D\to \bar{\nu}_e np}(x)$ are well defined. Thus, the procedure of the derivation of effective Lagrangians of low-energy nuclear transitions in the NNJL model treating the deuteron as a point-like particle coupled to nucleons and other particles through one-nucleon loop exchanges seems to be good established. We argue that the application of this procedure should get correct results for the derivation of effective Lagrangians of any other low-energy nuclear transitions, effective vertices, of nuclear reactions.

Some problems occur for the evaluation of the amplitudes of nuclear reactions demanding the continuation of matrix elements of low-energy nuclear transitions defined by the effective Lagrangians to the energy region far from thresholds.

The continuation of matrix elements of low-energy nuclear transitions demands in the NNJL model: 1) the spatial smearing of the NN pair in the $^1S_0$ state and 2) the spatial smearing of the deuteron caused by the finite value of the effective radius r_D. The spatial smearing of the NN pair in the $^1S_0$ state can be carried out by a summation of an infinite series of one-nucleon bubbles describing rescattering or differently a relative movement of the NN pair either in an initial or a final state of a nuclear reaction. The result of the NN rescattering can be expressed in terms of S wave scattering lengths and effective ranges in complete agreement with nuclear phenomenology of low-energy elastic NN scattering in the $^1S_0$ state. However, for the description of the spatial smearing of the deuteron the abilities of the NNJL model are resticted and most what one can do at present level of the development of the model is to introduce the spatial smearing of the deuteron phenomenologically in the form of the Fourier transform of the approximate $^3S_1$ wave function of the deuteron normalized to unity at $p = 0$: $F_D(p^2) = 1/(1 + r_D^2 p^2)$. We have chosen a simplest form of the spatial smearing of the deuteron. Of course, $F_D(p^2)$ can be taken in the more complicated form of the Fourier transform of the explicit wave function of the deuteron. Of course, such a dependence is not absolute and the spatial smearing of the deuteron can be taken into account in the form of phenomenological form factors as it has been done by Mintz [52], for example.

We would like to emphasize that the cross-section for the M1-capture $n + p \to D + \gamma$ for thermal neutrons has been calculated by accounting for the contributions of chiral one-meson loop corrections and the $\Delta(1232)$ resonance. The total cross-section for the M1-capture has been found dependent on the parameter Z defining the $\pi N \Delta$ coupling constant off-mass shell of the $\Delta(1232)$ resonance: $\sigma(np \to D\gamma)(T_n) = 325.5\,(1 + 0.246\,(1 - 2Z))^2\,\mathrm{mb}$ eq. (4.15). In order to fit the experimental value of the cross-section we should set $Z = 0.473$. This agrees with the experimental bound $|Z| \le 1/2$ [18]. At $Z = 1/2$

[5] This result is obtained at zero contribution of the nucleon tensor current (see appendix and the discussion in the conclusion).

the contribution of the $\Delta(1232)$ resonance to the amplitude of the M1-capture is defined only by the nucleon tensor current. Setting $Z = 1/2$ that is favoured theoretically [15] we calculate the cross-section for the M1-capture $\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}}) = 325.5\,\mathrm{mb}$ agreeing with the experimental data with accuracy better than 3%.

When matching our result for the cross-section for the M1-capture with the recent one obtained in the EFT approach by Chen, Rupak and Savage [9]: $\sigma(\mathrm{np} \to \mathrm{D}\gamma)(T_{\mathrm{n}}) = (287.1 + 6.51\,{}^{\not\pi}L_1)\,\mathrm{mb}$ (see eq. (3.49) of ref. [9]), we accentuate the dependence of the cross-section on the parameter ${}^{\not\pi}L_1$ undefined in the approach. This parameter has to be fixed from the experimental data [9]. In the NNJL model the parameter ${}^{\not\pi}L_1$ can be expressed in terms of the Z parameter as follows: ${}^{\not\pi}L_1 = 5.90 + 24.60\,(1-2Z) + 3.03\,(1-2Z)^2$. Thus, in the NNJL model the parameter ${}^{\not\pi}L_1$ acquires a distinct meaning of the contribution of the $\Delta(1232)$ resonance.

The obtained result for the M1-capture $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$ we have applied to the analysis of the photomagnetic disintegration of the deuteron $\gamma + \mathrm{D} \to \mathrm{n} + \mathrm{p}$. Due to time-reversal invariance the cross-section for the photomagnetic disintegration of the deuteron can be directly expressed through the cross-section for the M1-capture. We have compared the numerical values of the cross section $\sigma(\gamma\mathrm{D} \to \mathrm{np})(\omega)$ calculated in the NNJL model with the results obtained by Chen and Savage [10] and found a good agreement. Nevertheless, it should be emphasized that in the critical region of the photon energies $\omega \le 2\,\varepsilon_{\mathrm{D}} = 4.45\,\mathrm{MeV}$ restricting the energy region of the dominance of the photomagnetic disintegration of the deuteron the cross-section calculated in the NNJL model falls steeper than the cross-section obtained in the EFT. However, in this region the dominant role should be attributed to the E1-transition [10], which we have not considered. The comparison of our results on the photomagnetic disintegration of the deuteron with the experimental data demands the inclusion of the E1-transition as well. This is planned in our forthcoming publications.

The effective coupling constants of low-energy weak transitions $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$, $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\mathrm{e}}$, $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$, $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and $\nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{D} \to \nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{n} + \mathrm{p}$ have been found dependent on an arbitrary parameter $\bar{\xi}$ in the form $g_{\mathrm{V}} \to g_{\mathrm{V}}\,(1+\bar{\xi})$ caused by the contribution of the nucleon tensor current [1].

Since at $\bar{\xi} = 0$ we get the value of the astrophysical factor $S_{\mathrm{pp}}(0) = 4.08 \times 10^{-25}\,\mathrm{MeV\,b}$ in complete agreement with the recommended one $S_{\mathrm{pp}}(0) = 4.00 \times 10^{-25}\,\mathrm{MeV\,b}$ related to the Standard Solar Model [27,28], any non-zero value of $\bar{\xi}$ should lead to an Alternative Solar Model (ASM). The problem of the formulation of the ASM goes beyond the scope of this paper. Therefore, in order to dwell within the Standard Solar Model [27,28] we have to set $\bar{\xi} = 0$.

Setting $\bar{\xi} = 0$ we have shown that all low-energy weak nuclear reactions of astrophysical interest i) the solar proton burning $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$, ii) the pep-process $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\mathrm{e}}$ and iii) the disintegration of the deuteron by neutrinos and antineutrinos caused by charged $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$ and $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and neutral $\nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{D} \to \nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{n} + \mathrm{p}$ weak currents are described in the bulk in agreement with other theorical approaches and experimental data. The effective Lagrangians of low-energy weak nuclear transitions $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$, $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\mathrm{e}}$, $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$, $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and $\nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{D} \to \nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{n} + \mathrm{p}$ are determined by the anomaly of the one-nucleon loop triangle AAV-diagrams. This confirms the statement argued in the NNJL model concerning the dominant role of the one-nucleon loop anomalies for the description of low-energy nuclear forces within a quantum field theoretic approach.

We have shown that the contributions of low-energy elastic pp scattering in the ${}^1S_0$ state with the Coulomb repulsion to the amplitudes of the reactions $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$, $\nu_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$ and $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\mathrm{e}}$ are described in the NNJL model in full agreement with low-energy nuclear phenomenology in terms of the S wave scattering length and the effective range. The amplitude of low-energy elastic pp scattering has been obtained by summing up an infinite series of one-proton loop diagrams and evaluating the result of the summation at leading order in the large N_{C} expansion. The same method has been applied to the evaluation of the contribution of low-energy nuclear forces to the relative movements of the nn and np pairs, respectively, for the reactions $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and $\nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{D} \to \nu_{\mathrm{e}}(\bar{\nu}_{\mathrm{e}}) + \mathrm{n} + \mathrm{p}$. This has given the amplitudes of low-energy elastic nn and np scattering described in terms of S wave scattering lengths and effective ranges in agreement with low-energy nuclear phenomenology [26] as well.

The astrophysical factor $S_{\mathrm{pep}}(0)$ for the pep-process, $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\mathrm{e}}$, evaluated relative to $S_{\mathrm{pp}}(0)$ is found in full agreement with the result obtained by Bahcall and May [45].

The cross-sections for the antineutrino disintegration of the deuteron caused by charged $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and neutral $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \bar{\nu}_{\mathrm{e}} + \mathrm{n} + \mathrm{p}$ weak currents and averaged over the antineutrino spectrum $\langle\sigma^{\bar{\nu}_{\mathrm{e}}\mathrm{D} \to \mathrm{e}^+\mathrm{nn}}(E_{\bar{\nu}_{\mathrm{e}}})\rangle = 11.56 \times 10^{-45}\,\mathrm{cm}^2$ and $\langle\sigma^{\bar{\nu}_{\mathrm{e}}\mathrm{D} \to \bar{\nu}_{\mathrm{e}}\mathrm{np}}(E_{\bar{\nu}_{\mathrm{e}}})\rangle = 6.28 \times 10^{-45}\,\mathrm{cm}^2$ agree well with recent experimental data

$$\langle\sigma^{\bar{\nu}_{\mathrm{e}}D \to e^+\mathrm{nn}}(E_{\bar{\nu}_{\mathrm{e}}})\rangle_{\mathrm{exp}} = (9.83 \pm 2.04) \times 10^{-45}\,\mathrm{cm}^2,$$
$$\langle\sigma^{\bar{\nu}_{\mathrm{e}}D \to \bar{\nu}_{\mathrm{e}}\mathrm{np}}(E_{\bar{\nu}_{\mathrm{e}}})\rangle_{\mathrm{exp}} = (6.08 \pm 0.77) \times 10^{-45}\,\mathrm{cm}^2$$

obtained by the Reines's experimental group [36].

The cross-sections for the reactions $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and $\bar{\nu}_{\mathrm{e}} + \mathrm{D} \to \bar{\nu}_{\mathrm{e}} + \mathrm{n} + \mathrm{p}$ have been recently calculated by Butler and Chen [53] in the EFT approach. The obtained results have been written in the following general form $\sigma = (a + b\,L_{1,\mathrm{A}}) \times 10^{-42}\,\mathrm{cm}^2$ (see Table I of ref. [53]), where a and b are the parameters which have been calculated in the approach, whereas $L_{1,\mathrm{A}}$ is a free one. In the NNJL model the appearance of the free parameter is related to the contribution of the nucleon tensor current [1] that effectively leads to the change of the coupling constant $g_{\mathrm{V}} \to g_{\mathrm{V}}\,(1+\bar{\xi})$, where $\bar{\xi}$ is an arbitrary parameter. The best agreement with the recommended value of the astrophysical factor for the solar proton burning [29] and the contemporary experimental data [36] on the cross-sections for the antineutrino disintegration of the deuteron $\bar{\nu}_{\rm e}$ + D $\to$ e$^+$ + n + n and $\bar{\nu}_{\rm e}$ + D $\to$ $\bar{\nu}_{\rm e}$ + n + p caused by charged and neutral weak current, respectively, we obtain at $\bar{\xi} = 0$ (see appendix).

We would like to emphasize that the contribution of the $\Delta(1232)$ resonance to the amplitudes of the low-energy weak nuclear reactions p + p $\to$ D + e$^+$ + $\nu_{\rm e}$, p + e$^-$ + p $\to$ $\nu_{\rm e}$ + D, $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p, $\bar{\nu}_{\rm e}$ + D $\to$ e$^+$ + n + n and $\bar{\nu}_{\rm e}$ + D $\to$ $\bar{\nu}_{\rm e}$ + n + p can be neglected. In fact, the contribution of the $\Delta(1232)$ resonance to the amplitudes of these reactions is of order of the momentum of the leptonic pair. This is due to the gauge invariance of the effective interactions $\Delta N W^+$ and $\Delta N Z$ which should be proportional to $W^+_{\mu\nu} = \partial_\mu W^+_\nu - \partial_\nu W^+_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, respectively. Since the amplitudes of the low-energy weak nuclear reactions under consideration are defined by the Gamow-Teller transitions, the terms proportional to the momentum of the leptonic pair give negligible small contributions. Thus, the cross-sections for low-energy weak nuclear reactions enumerated above do not depend practically on the uncertainties of the parameter Z.

This distinguishes low-energy weak nuclear reactions with the deuteron from the neutron-proton radiative capture analysed in the NNJL model. Unlike the low-energy weak nuclear reactions the contribution of the $\Delta(1232)$ resonance to the amplitude of the neutron-proton radiative capture is essential for the explanation of the experimental data.

The cross-section for the reaction $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p has been evaluated with respect to $S_{\rm pp}(0)$. We have found an enhancement of the cross-section by a factor of 1.6 on the average relative to the results obtained in the PMA. It would be important to verify this result for the reaction $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p in solar neutrino experiments planned by SNO. Indeed, first, this should provide an experimental study of $S_{\rm pp}(0)$ and, second, the cross-sections for the antineutrino disintegration of the deuteron caused by charged $\bar{\nu}_{\rm e}$ + D $\to$ e$^+$ + n + n and neutral $\bar{\nu}_{\rm e}$ + D $\to$ $\bar{\nu}_{\rm e}$ + n + p weak currents have been found in good agreement with recent experimental data [36].

For the comparison of the cross-sections for the reactions $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p and $\nu_{\rm e}$ + D $\to$ $\nu_{\rm e}$ + n + p caused by charged and neutral weak currents, respectively, we have averaged the cross-sections over the ^{8}B solar neutrino energy spectrum at energy region $5\,{\rm MeV} \le E_{\nu_{\rm e}} \le 15\,{\rm MeV}$, where the lower bound is related to the experimental threshold of the experiments at SNO and the upper one is defined by the kinematics of the β decay $^8{\rm B} \to {}^8{\rm Be}^* + {\rm e}^+ + \nu_{\rm e}$ being the source of the ^{8}B neutrinos in the solar core. We have obtained

$$\langle \sigma^{\nu_{\rm e}{\rm D}\to{\rm e}^-{\rm pp}}(E_{\nu_{\rm e}})\rangle_{\Phi(^8{\rm B})} = 2.62\times 10^{-42}\,{\rm cm}^2,$$
$$\langle \sigma^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(E_{\nu_{\rm e}})\rangle_{\Phi(^8{\rm B})} = 1.85\times 10^{-43}\,{\rm cm}^2.$$

The experimental value for the cross-section for the reaction $\nu_{\rm e}$ + D $\to$ e$^-$ + p + p caused by the charged weak current can, in principle, differ from the theoretical one due to a possible contribution of the neutrino flavour oscillations [27,48]. In turn, the averaged value of the cross-section for the reaction $\nu_{\rm e}$ + D $\to$ $\nu_{\rm e}$ + n + p caused by the neutral weak current should be directly compared with the experimental data, since it should not depend on whether neutrino flavours oscillate or not. Of course, the former is valid only if there is no so-called *sterile* neutrino [27,48] having no interactions with Standard Model particles [39].

Concluding the paper we would like to emphasize that the NNJL model conveys the idea of a dominant role of one-fermion loop (one-nucleon loop) anomalies from elementary particle physics to the nuclear one. This is a new approach to the description of low-energy nuclear forces in physics of light nuclei. In spite of the fact that almost 30 years have passed after the discovery of one-fermion loop anomalies and application of these anomalies to the evaluation of effective chiral Lagrangians of low-energy interactions of hadrons, in nuclear physics fermion-loop anomalies have not been applied to the analysis of low-energy nuclear interactions and properties of light nuclei. The contributions of one-nucleon loop anomalies are strongly related to high-energy N$\bar{\rm N}$ fluctuations of virtual nucleon fields [1,54]. An important role of N$\bar{\rm N}$ fluctuations for the correct description of low-energy properties of finite nuclei has been understood in ref. [55]. Moreover, N$\bar{\rm N}$ fluctuations have been described in terms of one-nucleon loop diagrams within quantum field theoretic approaches, but the contributions of one-nucleon loop anomalies have not been considered. The NNJL model allows to fill this blank. Within the framework of the NNJL model we aim to understand, in principle, the role of nucleon-loop anomalies for the description of a dynamics of low-energy nuclear forces at the quantum field theoretic level.

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Appendix A. The effective Lagrangian of the transition p + p $\to$ D + e$^+$ + $\nu_{\rm e}$

The reaction p + p $\to$ D + e$^+$ + $\nu_{\rm e}$ runs through the intermediate W-boson exchange, p + p $\to$ D + W$^+$ $\to$ D + e$^+$ + $\nu_{\rm e}$. In the NNJL model we determine this transition in terms of the following effective interactions [1,39]:

$$\mathcal{L}_{\rm npD}(x) = -ig_{\rm V}[\bar{p^c}(x)\gamma^{\mu}n(x) - \bar{n^c}(x)\gamma^{\mu}p(x)]\,D^{\dagger}_{\mu}(x),$$
$$\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x) = \frac{1}{2}\,C_{\rm NN}\,\{[\bar{p}(x)\,\gamma^{\mu}\gamma^5 p^c(x)]\,[\bar{p^c}(x)\,\gamma_{\mu}\gamma^5 p(x)]$$
$$+[\bar{p}(x)\,\gamma^5 p^c(x)]\,[\bar{p^c}(x)\,\gamma^5 p(x)]\},$$
$$\mathcal{L}_{\rm npW}(x) = -\frac{g_{\rm W}}{2\sqrt{2}}\,\cos\vartheta_{\rm C}\,[\bar{n}(x)\gamma^{\nu}(1-g_{\rm A}\gamma^5)p(x)]\,W^-_{\nu}(x). \tag{A.1}$$

The transition $W^+ \to e^+ + \nu_e$ is defined by the Lagrangian [39]

$$\mathcal{L}_{\nu_e e^+ W}(x) = -\frac{g_{\rm W}}{2\sqrt{2}}\,[\bar{\psi}_{\nu_e}(x)\gamma^{\nu}(1-\gamma^5)\psi_{\rm e}(x)]\,W^+_{\nu}(x). \tag{A.2}$$

The electroweak coupling constant $g_{\rm W}$ is connected with the Fermi weak constant $G_{\rm F}$ and the mass of the W-boson $M_{\rm W}$ through the relation [39]

$$\frac{g^2_{\rm W}}{8M^2_{\rm W}} = \frac{G_{\rm F}}{\sqrt{2}}. \tag{A.3}$$

For the evaluation of the effective Lagrangian $\mathcal{L}^{\rm pp\to De^+\nu_e}_{\rm eff}(x)$ it is convenient to use the interaction

$$\mathcal{L}_{\rm npW}(x) = [\bar{n}(x)\gamma^{\nu}(1-g_{\rm A}\gamma^5)p(x)]\,W^-_{\nu}(x) \tag{A.4}$$

and for the description of the subsequent weak transition $W^+ \to e^+ + \nu_e$ to replace the operator of the W-boson field by the operator of the leptonic weak current

$$W^-_{\nu}(x) \to -\frac{G_{\rm V}}{\sqrt{2}}\,[\bar{\psi}_{\nu_e}(x)\gamma_{\nu}(1-\gamma^5)\psi_{\rm e}(x)], \tag{A.5}$$

where $G_{\rm V} = G_{\rm F}\,\cos\vartheta_{\rm C}$.

The S matrix describing the transitions like $p + p \to D + W^+$ is defined by

$$S = {\rm T}e^{i\int d^4x\,[\mathcal{L}_{\rm npD}(x) + \mathcal{L}_{\rm npW}(x) + \mathcal{L}^{\rm pp\to pp}_{\rm eff}(x) + \ldots]} \tag{A.6}$$

where T is the time-ordering operator and the ellipses denote the contribution of interactions irrelevant for the problem.

For the evaluation of the effective Lagrangian $\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x)$ we have to consider the third-order term of the S-matrix which reads

$$S^{(3)} = \frac{i^3}{3!}\int d^4x_1 d^4x_2 d^4x_3\,{\rm T}([\mathcal{L}_{\rm npD}(x_1) + \mathcal{L}_{\rm npW}(x_1)$$
$$+\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1) + \ldots]\,[\mathcal{L}_{\rm npD}(x_2) + \mathcal{L}_{\rm npW}(x_2)$$
$$+\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_2) + \ldots]\,[\mathcal{L}_{\rm npD}(x_3) + \mathcal{L}_{\rm npW}(x_3)$$
$$+\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_3) + \ldots]) = -i\int d^4x_1 d^4x_2 d^4x_3$$
$$\times{\rm T}(\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3)) + \ldots \tag{A.7}$$

The ellipses denote the terms which do not contribute to the transition $p + p \to D + W^+$ and the interaction $\mathcal{L}_{\rm npW}(x)$. The S-matrix element $S^{(3)}_{\rm pp\to DW^+}$ describing the transition $p + p \to D + W^+$ we determine as follows:

$$S^{(3)}_{\rm pp\to DW^+} =$$
$$-i\int d^4x_1 d^4x_2 d^4x_3\,{\rm T}(\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3)). \tag{A.8}$$

For the derivation of the effective Lagrangian $\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x)$ from the S-matrix element eq. (A.8) we should make all necessary contractions of the operators of the proton and the neutron fields. These contractions we denote by the brackets as

$$\langle S^{(3)}_{\rm pp\to DW^+}\rangle = -i\int d^4x_1 d^4x_2 d^4x_3 \times$$
$$\langle{\rm T}(\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3))\rangle. \tag{A.9}$$

Now the effective Lagrangian $\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x)$ related to the S-matrix element $\langle S^{(3)}_{\rm pp\to DW^+}\rangle$ can be defined by

$$\langle S^{(3)}_{\rm pp\to DW^+}\rangle = i\int d^4x\,\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x) =$$
$$-i\int d^4x_1 d^4x_2 d^4x_3\,\langle{\rm T}(\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3))\rangle. \tag{A.10}$$

In terms of the operators of the interacting fields the effective Lagrangian $\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x)$ reads

$$\int d^4x\,\mathcal{L}^{\rm pp\to DW^+}_{\rm eff}(x) =$$
$$-\int d^4x_1 d^4x_2 d^4x_3\langle{\rm T}(\mathcal{L}^{\rm pp\to pp}_{\rm eff}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3))\rangle =$$
$$-\frac{1}{2}\,C_{\rm NN}\,\times\,(-ig_{\rm V})\,\times\,(-g_{\rm A})$$
$$\times\int d^4x_1 d^4x_2 d^4x_3\,{\rm T}([\bar{p^c}(x_1)\,\gamma_{\alpha}\gamma^5 p(x_1)]\,D^{\dagger}_{\mu}(x_2)\,W^-_{\nu}(x_3))$$
$$\times\langle 0|{\rm T}([\bar{p}(x_1)\gamma^{\alpha}\gamma^5 p^c(x_1)][\bar{p^c}(x_2)\gamma^{\mu}n(x_2) - \bar{n^c}(x_2)\gamma^{\mu}p(x_2)]$$
$$\times[\bar{n}(x_3)\gamma^{\nu}\gamma^5 p(x_3)])|0\rangle - \frac{1}{2}\,C_{\rm NN}\,\times\,(-ig_{\rm V})\,\times\,(-g_{\rm A})$$
$$\times\int d^4x_1 d^4x_2 d^4x_3\,{\rm T}([\bar{p^c}(x_1)\gamma^5 p(x_1)]\,D^{\dagger}_{\mu}(x_2)\,W^-_{\nu}(x_3))$$
$$\times\langle 0|{\rm T}([\bar{p}(x_1)\gamma^5 p^c(x_1)][\bar{p^c}(x_2)\gamma^{\mu}n(x_2) - \bar{n^c}(x_2)\gamma^{\mu}p(x_2)]$$
$$\times[\bar{n}(x_3)\gamma^{\nu}\gamma^5 p(x_3)])|0\rangle. \tag{A.11}$$

Since $p + p \to D + W^+$ is the Gamow-Teller transition, we have taken into account the W-boson coupled to the axial-vector nucleon current.

Due to the relation $\bar{n^c}(x_2)\gamma^\mu p(x_2) = -\bar{p^c}(x_2)\gamma^\mu n(x_2)$ the r.h.s. of eq. (A.11) can be reduced as follows

$$\int d^4x\,\mathcal{L}_{\rm eff}^{\rm pp\to DW^+}(x) = -\int d^4x_1 d^4x_2 d^4x_3 \times\langle {\rm T}(\mathcal{L}_{\rm eff}^{\rm pp\to pp}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3))\rangle = \times C_{\rm NN}\times(-ig_{\rm V})\times g_{\rm A}\int d^4x_1 d^4x_2 d^4x_3 \times{\rm T}([\bar{p^c}(x_1)\gamma_\alpha\gamma^5 p(x_1)]D^\dagger_\mu(x_2)W^-_\nu(x_3))\langle 0|{\rm T}([\bar{p}(x_1)\gamma^\alpha\gamma^5 p^c(x_1)] \times[\bar{p^c}(x_2)\gamma^\mu n(x_2)][\bar{n}(x_3)\gamma^\nu\gamma^5 p(x_3)])|0\rangle + C_{\rm NN}\times(-ig_{\rm V}) \times g_{\rm A}\int d^4x_1 d^4x_2 d^4x_3\,{\rm T}([\bar{p^c}(x_1)\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2)\,W^-_\nu(x_3)) \times\langle 0|{\rm T}([\bar{p}(x_1)\gamma^5 p^c(x_1)][\bar{p^c}(x_2)\gamma^\mu n(x_2)][\bar{n}(x_3)\gamma^\nu\gamma^5 p(x_3)])|0\rangle. \quad ({\rm A.12})$$

Making the necessary contractions we arrive at the expression

$$\int d^4x\,\mathcal{L}_{\rm eff}^{\rm pp\to DW^+}(x) = -\int d^4x_1 d^4x_2 d^4x_3 \langle {\rm T}(\mathcal{L}_{\rm eff}^{\rm pp\to pp}(x_1)\mathcal{L}_{\rm npD}(x_2)\mathcal{L}_{\rm npW}(x_3))\rangle = 2\times C_{\rm NN}\times(-ig_{\rm V})\times g_{\rm A}\int d^4x_1 d^4x_2 d^4x_3 \times{\rm T}([\bar{p^c}(x_1)\,\gamma_\alpha\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2)\,W^-_\nu(x_3)) \times(-1)\,{\rm tr}\{\gamma^\alpha\gamma^5(-i)S^c_F(x_1-x_2)\gamma^\mu(-i) \times S_F(x_2-x_3)\gamma^\nu\gamma^5(-i)S_F(x_3-x_1)\} +2\times C_{\rm NN}\times(-ig_{\rm V})\times g_{\rm A}\int d^4x_1 d^4x_2 d^4x_3 \times{\rm T}([\bar{p^c}(x_1)\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2)\,W^-_\nu(x_3)) \times(-1)\,{\rm tr}\{\gamma^5(-i)S^c_F(x_1-x_2)\gamma^\mu(-i) \times S_F(x_2-x_3)\gamma^\nu\gamma^5(-i)S_F(x_3-x_1)\}, \quad ({\rm A.13})$$

where the combinatorial factor 2 takes into account the fact that the protons are identical particles in the nucleon loop.

In the momentum representation of the Green functions we get

$$\int d^4x\,\mathcal{L}_{\rm eff}^{\rm pp\to DW^+}(x) = -i\,g_{\rm A}C_{\rm NN}\frac{g_{\rm V}}{8\pi^2}\int d^4x_1\int\frac{d^4x_2 d^4k_2}{(2\pi)^4}\frac{d^4x_3 d^4k_3}{(2\pi)^4} \times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)} \times{\rm T}([\bar{p^c}(x_1)\,\gamma_\alpha\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2)\,W^-_\nu(x_3))\,\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) -i\,g_{\rm A}C_{\rm NN}\frac{g_{\rm V}}{8\pi^2}\int d^4x_1\int\frac{d^4x_2 d^4k_2}{(2\pi)^4}\frac{d^4x_3 d^4k_3}{(2\pi)^4} \times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)} \times{\rm T}([\bar{p^c}(x_1)\,\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2)\,W^-_\nu(x_3))\;\mathcal{J}^{\mu\nu}(k_2,k_3;Q). \quad ({\rm A.14})$$

The structure functions $\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q)$ and $\mathcal{J}^{\mu\nu}(k_2,k_3;Q)$ are defined by the momentum integrals

see equations (A.15) *on next page*

We have introduced a 4-vector $Q = a\,k_2 + b\,k_3$ caused by an arbitrary shift of a virtual momentum with arbitrary parameters a and b.

In order to obtain the effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$ of the transition $\rm p + p \to D + e^+ + \nu_e$ we have to replace the operator of the W-boson field by the operator of the leptonic weak current eq. (A.5):

$$\int d^4x\,\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x) = i\,g_{\rm A}C_{\rm NN}\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm V}}{8\pi^2}\int d^4x_1\int\frac{d^4x_2 d^4k_2}{(2\pi)^4}\frac{d^4x_3 d^4k_3}{(2\pi)^4} \times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)} \times{\rm T}([\bar{p^c}(x_1)\,\gamma_\alpha\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2) \times[\bar{\psi}_{\nu_e}(x_3)\gamma_\nu(1-\gamma^5)\psi_e(x_3)])\,\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) +i\,g_{\rm A}C_{\rm NN}\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm V}}{8\pi^2}\int d^4x_1\int\frac{d^4x_2 d^4k_2}{(2\pi)^4}\frac{d^4x_3 d^4k_3}{(2\pi)^4} \times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)} \times{\rm T}([\bar{p^c}(x_1)\,\gamma^5 p(x_1)]\,D^\dagger_\mu(x_2) \times[\bar{\psi}_{\nu_e}(x_3)\gamma_\nu(1-\gamma^5)\psi_e(x_3)])\,\mathcal{J}^{\mu\nu}(k_2,k_3;Q). \quad ({\rm A.16})$$

Thus, the problem of the evaluation of the effective Lagrangian of the transition $\rm p + p \to D + e^+ + \nu_e$ reduces itself to the problem of the evaluation of the structure functions eq. (A.15). The momentum k_3 is related to the 4-momentum of the leptonic pair. Due to the Gamow-Teller type of the transition $\rm p + p \to D + W^+$ the contribution proportional to the 4-momentum of the leptonic pair turns out to be much smaller with respect to the contribution proportional to the 4-momentum of the deuteron k_2. Therefore, without loss of generality we can set $k_3 = 0$ in the integrand. This gives

see equations (A.17) *on next page*

The evaluation of the momentum integrals at leading order in the $1/M_{\rm N}$ expansion corresponding the leading order in the large $N_{\rm C}$ expansion due to the proportionality $M_{\rm N}\sim N_{\rm C}$ in QCD with $SU(N_{\rm C})$ gauge group at $N_{\rm C}\to\infty$ [22] (see also ref. [1]) yields

$$\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) = 3\,(k_2^\alpha g^{\nu\mu} - k_2^\nu g^{\mu\alpha}) +\frac{1}{9}\,(1+2a)\,(k_2^\alpha g^{\nu\mu} + k_2^\nu g^{\mu\alpha}), \mathcal{J}^{\mu\nu}(k_2,k_3;Q) = g^{\mu\nu}4\,M_{\rm N}\,J_2(M_{\rm N}) \sim O(1/N_C^2), \quad ({\rm A.18})$$

where the terms proportional to k_2^μ have been dropped, because they produce the contributions to the effective Lagrangian multiplied by $\partial^\mu D_\mu(x)$ vanishing by virtue of the constraint $\partial^\mu D_\mu(x) = 0$. Then, $J_2(M_{\rm N})$ is a logarithmically divergent integral defined in the NNJL model in terms of the cut-off $\Lambda_D = 46.172\,\text{MeV}$ such as $\Lambda_{\rm D} \ll M_{\rm N}$

$$\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) = \int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^\alpha\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\gamma^\mu\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\nu\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}-\hat{k}_3}\Big\},$$
$$\mathcal{J}^{\mu\nu}(k_2,k_3;Q) = \int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\gamma^\mu\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\nu\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}-\hat{k}_3}\Big\}. \quad (A.15)$$

$$\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) = \int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^\alpha\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\gamma^\mu\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\nu\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}}\Big\},$$
$$\mathcal{J}^{\mu\nu}(k_2,k_3;Q) = \int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\gamma^\mu\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\nu\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}}\Big\}, \quad (A.17)$$

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[1]:

$$J_2(M_N) = \int\frac{d^4k}{\pi^2 i}\frac{1}{(M_N^2-k^2)^2} = 2\int_0^{\Lambda_D}\frac{d|\mathbf{k}|\mathbf{k}^2}{(M_N^2+\mathbf{k}^2)^{3/2}} = \frac{2}{3}\left(\frac{\Lambda_D}{M_N}\right)^3 \sim O(1/N_C^3). \quad (A.19)$$

The cut-off Λ_D restricts 3-momenta of the virtual nucleon fluctuations forming the physical deuteron [1]. Due to the uncertainty relation $\Delta r\,\Lambda_D \sim 1$ the spatial region of virtual nucleon fluctuations forming the physical deuteron is defined by $\Delta r \sim 4.274\,\mathrm{fm}$. This agrees with the effective radius of the deuteron $r_D = 1/\sqrt{\varepsilon_D M_N} = 4.319\,\mathrm{fm}$.

Keeping the terms of the same order in the large N_C expansion we get the structure functions

$$\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) = 3\,(k_2^\alpha g^{\nu\mu} - k_2^\nu g^{\mu\alpha}) + \frac{1}{9}\,(1+2a)\,(k_2^\alpha g^{\nu\mu} + k_2^\nu g^{\mu\alpha}),$$
$$\mathcal{J}^{\mu\nu}(k_2,k_3;Q) = 0, \quad (A.20)$$

The structure functions eq. (A.20) define the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x)$:

$$\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x) = g_A C_{NN}\frac{G_V}{\sqrt{2}}\frac{3g_V}{8\pi^2}\,[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)] \times\Big[(\partial_\mu D^\dagger_\nu(x) - \partial_\nu D^\dagger_\mu(x)) + \frac{1}{27}(1+2a)(\partial_\mu D^\dagger_\nu(x) + \partial_\nu D^\dagger_\mu(x))\Big]. \quad (A.21)$$

In order to remove uncertainty caused by the shift of virtual momenta of the one-nucleon loop diagrams we demand the invariance of the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x)$ under gauge transformations of the deuteron field

$$D^\dagger_\mu(x) \to D^\dagger_\mu(x) + \partial_\mu\Omega(x), \quad (A.22)$$

where $\Omega(x)$ is a gauge function. The requirement of the invariance of the effective Lagrangian of the low-energy transition $p + p \to D + e^+ + \nu_e$ under gauge transformation eq. (A.22) imposes the constraint $a = -1/2$. This reduces the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x)$ to the form

$$\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x) = g_A C_{NN}\frac{G_V}{\sqrt{2}}\frac{3g_V}{8\pi^2} \times D^\dagger_{\mu\nu}(x)\,[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)]. \quad (A.23)$$

This effective Lagrangian is defined by the structure function $\mathcal{J}^{\alpha\mu\nu}(k_2,k_2;Q)$:

$$\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q) = 3\,(k_2^\alpha g^{\nu\mu} - k_2^\nu g^{\mu\alpha}). \quad (A.24)$$

The structure function $\mathcal{J}^{\alpha\mu\nu}(k_2,k_2;Q)$ does not depend on the mass of virtual nucleons and according to Gertsein and Jackiw [37] can be valued as the anomaly of the one-nucleon triangle AAV-diagram. The requirement of gauge invariance applied to remove ambiguities of the structure function $\mathcal{J}^{\alpha\mu\nu}(k_2,k_3;Q)$ and to fix the contribution of the anomaly of the one-nucleon loop AAV-diagrams is in complete agreement with the derivation of the Adler-Bell-Jackiw axial-vector anomaly performed in terms of one-nucleon loop AVV-diagrams (see *Jackiw* [54]).

The effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{\bar{\nu}_e D\to e^+ nn}(x)$ describing the low-energy transition $\bar{\nu}_e + D \to e^+ + n + n$ can be obtained by the way analogous to $\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x)$ and reads

$$\mathcal{L}_{\mathrm{eff}}^{\bar{\nu}_e D\to e^+ nn}(x) = -\,g_A\,C_{NN}\frac{G_V}{\sqrt{2}}\frac{3g_V}{8\pi^2} \times D_{\mu\nu}(x)\,[\bar{n}(x)\gamma^\mu\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)]. \quad (A.25)$$

In the low-energy limit when $D^\dagger_{\mu\nu}(x)\,[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)] \to -\,2\,i\,M_N D^\dagger_\nu(x)[\bar{p^c}(x)\gamma^5 p(x)]$ the effective Lagrangian eq. (A.23) reduces itself to the form

$$\mathcal{L}_{\mathrm{eff}}^{pp\to De^+\nu_e}(x) = -\,i\,g_A\,M_N\,C_{NN}\frac{G_V}{\sqrt{2}}\frac{3g_V}{4\pi^2} \times D^\dagger_\nu(x)\,[\bar{p^c}(x)\gamma^5 p(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)]. \quad (A.26)$$

The low-energy reduction $D_{\mu\nu}[\bar{n}(x)\gamma^\mu\gamma^5 n^c(x)] \to -\,2\,i\,M_N D_\nu(x)\bar{n}(x)\gamma^5 n^c(x)$ applied to the effective

Lagrangian $\mathcal{L}_{\rm eff}^{\bar{\nu}_{\rm e}{\rm D}\to {\rm e}^+{\rm nn}}(x)$ gives

$$\mathcal{L}_{\rm eff}^{\bar{\nu}_{\rm e}{\rm D}\to {\rm e}^+{\rm nn}}(x) = i\,g_{\rm A}\,M_{\rm N}\,C_{\rm NN}\,\frac{G_{\rm V}}{\sqrt{2}}\frac{3g_{\rm V}}{4\pi^2} \times D_\nu(x)\,[\bar{n}(x)\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\rm e}(x)]. \quad ({\rm A.27})$$

The effective Lagrangians eq. (A.23) and eq. (A.25) as well as eq. (A.26) and eq. (A.27) testify distinctly that the transitions p + p → D + e^+ + $\nu_{\rm e}$ and $\bar{\nu}_{\rm e}$ + D → e^+ n + n are governed by the same dynamics of low-energy nuclear forces in agreement with charge independence of weak interaction strength.

For the evaluation of the effective Lagrangian $\mathcal{L}_{\rm eff}^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(x)$ of the transition $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p in the NNJL model one should use the following Lagrangians:

$$\begin{aligned}
&\mathcal{L}^\dagger_{\rm npD}(x) = -ig_{\rm V}[\bar{p}(x)\gamma^\mu n^c(x) - \bar{n}(x)\gamma^\mu p^c(x)]\,D_\mu(x),\\
&\mathcal{L}_{\rm eff}^{\rm np\to np}(x) = C_{\rm NN}\,\{[\bar{p}(x)\,\gamma^\mu\gamma^5 n^c(x)]\,[\bar{n^c}(x)\,\gamma_\mu\gamma^5 p(x)]\\
&+[\bar{p}(x)\,\gamma^5 n^c(x)]\,[\bar{n^c}(x)\,\gamma^5 p(x)]\},\\
&\mathcal{L}_{\rm NNZ}(x) = g_{\rm A}\,[\bar{p}(x)\gamma^\nu\gamma^5 p(x) - \bar{n}(x)\gamma^\nu\gamma^5 n(x)]\,Z_\nu(x),\\
&Z_\nu(x) \to \frac{G_{\rm F}}{2\sqrt{2}}\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\rm e}(x)]. \qquad ({\rm A.28})
\end{aligned}$$

The effective Lagrangian $\mathcal{L}_{\rm eff}^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(x)$ of the low-energy transition $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p is then defined by

$$\mathcal{L}_{\rm eff}^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(x) = -\,g_{\rm A}\,C_{\rm NN}\,\frac{G_{\rm F}}{\sqrt{2}}\frac{3g_{\rm V}}{8\pi^2}D_{\mu\nu}(x) \times[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\nu_{\rm e}}(x)]. \quad ({\rm A.29})$$

In the low-energy limit when $D_{\mu\nu}(x)[\bar{p}(x)\gamma^\mu\gamma^5 n^c(x)] \to -2iM_{\rm N}D_\nu(x)[\bar{p}(x)\gamma^5 n^c(x)]$ the effective Lagrangian reduces itself to the from

$$\mathcal{L}_{\rm eff}^{\nu_{\rm e}{\rm D}\to\nu_{\rm e}{\rm np}}(x) = i\,g_{\rm A}\,M_{\rm N}\,C_{\rm NN}\,\frac{G_{\rm F}}{\sqrt{2}}\frac{3g_{\rm V}}{4\pi^2} \times D_\nu(x)\,[\bar{p}(x)\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\nu_{\rm e}}(x)]. \quad ({\rm A.30})$$

For the evaluation of the matrix element of the transition $\nu_{\rm e}$ + D → $\nu_{\rm e}$ + n + p one would use the wave function of the np pair in the standard form $|n(p_1)p(p_2)\rangle = a^\dagger_{\rm n}(p_1,\sigma_1)\,a^\dagger_{\rm p}(p_2,\sigma_2)|0\rangle$. The contribution of low-energy nuclear forces to the relative movement of the np pair in the ${}^1S_0$ state should be described by the infinite series of one-nucleon bubbles evaluated at leading order in the large $N_{\rm C}$ expansion. The result should be expressed in terms of the S wave scattering length $a_{\rm np}$ and the effective range $r_{\rm np}$ of low-energy elastic np scattering in the ${}^1S_0$ state.

Now let us obtain the contribution of the nucleon tensor current eq. (4.10). In terms of the structure functions the effective Lagrangian $\delta\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$

$$\begin{aligned}
&\int {\rm d}^4x\,\delta\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x) =\\
&= -\,g_{\rm A}\frac{C_{\rm NN}}{8\pi^2}\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm T}}{2M_{\rm N}}\int {\rm d}^4x_1\int\frac{{\rm d}^4x_2{\rm d}^4k_2}{(2\pi)^4}\frac{{\rm d}^4x_3{\rm d}^4k_3}{(2\pi)^4}\\
&\times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)}\\
&\times {\rm T}([\bar{p^c}(x_1)\,\gamma_\alpha\gamma^5 p(x_1)]\,D^\dagger_{\mu\nu}(x_2)\\
&\times[\bar{\psi}_{\nu_{\rm e}}(x_3)\gamma_\lambda(1-\gamma^5)\psi_{\rm e}(x_3)])\,J^{\alpha\mu\nu\lambda}(k_2,k_3;Q)\\
&-\,g_{\rm A}\frac{C_{\rm NN}}{8\pi^2}\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm T}}{2M_{\rm N}}\int {\rm d}^4x_1\int\frac{{\rm d}^4x_2{\rm d}^4k_2}{(2\pi)^4}\frac{{\rm d}^4x_3{\rm d}^4k_3}{(2\pi)^4}\\
&\times e^{-ik_2\cdot(x_2-x_1)}e^{-ik_3\cdot(x_3-x_1)}\\
&\times {\rm T}([\bar{p^c}(x_1)\,\gamma^5 p(x_1)]\,D^\dagger_{\mu\nu}(x_2)\\
&\times[\bar{\psi}_{\nu_{\rm e}}(x_3)\gamma_\lambda(1-\gamma^5)\psi_{\rm e}(x_3)])\,J^{\mu\nu\lambda}(k_2,k_3;Q). \qquad ({\rm A.31})
\end{aligned}$$

The structure functions are given by

see equations (A.32) *on next page*

where a 4-vector $Q = a\,k_2 + b\,k_3$ is an arbitrary shift of a virtual momentum with arbitrary parameters a and b.

The evaluation of the structure functions eq. (A.32) at leading order in the large $N_{\rm C}$ expansion gives the following effective Lagrangian $\delta\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x)$ caused by the contribution of the nucleon tensor current

$$\begin{aligned}
&\delta\mathcal{L}_{\rm eff}^{\rm pp\to De^+\nu_e}(x) = g_{\rm A}\,C_{\rm NN}\,\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm T}}{2\pi^2}\\
&\times\Big\{D^\dagger_{\mu\nu}(x)\,[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)] - \frac{ia}{2M_{\rm N}}\,\partial^\mu D^\dagger_{\mu\nu}(x)[\bar{p^c}(x)\gamma^5 p(x)]\Big\}\\
&\times[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\rm e}(x)]. \qquad ({\rm A.33})
\end{aligned}$$

It is seen that the coupling constants of the effective Lagrangian depend on the arbitrary parameter a caused by a shift of a virtual momentum.

The analogous expression one can get for the effective Lagrangian of the transition $\bar{\nu}_{\rm e}$ + D → e^+ + n + n caused by the contribution of the nucleon tensor current as well

$$\begin{aligned}
&\delta\mathcal{L}_{\rm eff}^{\bar{\nu}_{\rm e}{\rm D}\to {\rm e}^+{\rm nn}}(x) = g_{\rm A}\,C_{\rm NN}\,\frac{G_{\rm V}}{\sqrt{2}}\frac{g_{\rm T}}{2\pi^2}\Big\{D_{\mu\nu}(x)\\
&\times[\bar{n}(x)\gamma^\mu\gamma^5 n^c(x)] - \frac{ia}{2M_{\rm N}}\,\partial^\mu D_{\mu\nu}(x)[\bar{n}(x)\gamma^5 n^c(x)]\Big\}\\
&\times[\bar{\psi}_{\nu_{\rm e}}(x)\gamma^\nu(1-\gamma^5)\psi_{\rm e}(x)]. \qquad ({\rm A.34})
\end{aligned}$$

In the low-energy limit when

$$\begin{aligned}
&D^\dagger_{\mu\nu}(x)\,[\bar{p^c}(x)\gamma^\mu\gamma^5 p(x)] \to -\,2\,i\,M_{\rm N}D^\dagger_\nu(x)\,[\bar{p^c}(x)\gamma^5 p(x)],\\
&D_{\mu\nu}(x)\,[\bar{n}(x)\gamma^\mu\gamma^5 n^c(x)] \to -\,2\,i\,M_{\rm N}D_\nu(x)\,[\bar{n}(x)\gamma^5 n(x)] \qquad ({\rm A.35})
\end{aligned}$$

$$\mathcal{J}^{\alpha\mu\nu\lambda}(k_2,k_3;Q) = \int \frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^\alpha\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\sigma^{\mu\nu}\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\lambda\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}-\hat{k}_3}\Big\},$$
$$\mathcal{J}^{\mu\nu\lambda}(k_2,k_3;Q) = \int \frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}+\hat{k}_2}\sigma^{\mu\nu}\frac{1}{M_N-\hat{k}-\hat{Q}}\gamma^\lambda\gamma^5\frac{1}{M_N-\hat{k}-\hat{Q}-\hat{k}_3}\Big\}, \quad (A.32)$$

the effective Lagrangians eq. (A.33) and eq. (A.34) can be recast into the form

$$\delta\mathcal{L}^{pp\to De^+\nu_e}_{eff}(x) = -\,i\,g_A\,M_N\,C_{NN}$$
$$\times\frac{G_V}{\sqrt{2}}\frac{3g_T}{4\pi^2}\,\xi\,D^\dagger_\nu(x)[\bar{p^c}(x)\gamma^5 p(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)],$$
$$\delta\mathcal{L}^{\bar{\nu}_e D\to e^+nn}_{eff}(x) = -\,i\,g_A\,M_N\,C_{NN}$$
$$\times\frac{G_V}{\sqrt{2}}\frac{3g_T}{4\pi^2}\,\xi\,D_\nu(x)[\bar{n}(x)\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)], \quad (A.36)$$

where ξ is an arbitrary parameter related to the parameter a as follows:

$$\xi = \frac{1}{3}\,(4\,-\,a). \quad (A.37)$$

The total effective Lagrangians of the transitions p + p → D + e^+ + ν_e, $\bar{\nu}_e$ + D → e^+ + n + n and $\bar{\nu}_e$ + D → $\bar{\nu}_e$ + n + p are defined by

$$\mathcal{L}^{pp\to De^+\nu_e}_{eff,tc}(x) = -\,i\,(1+\bar{\xi})\,g_A\,M_N\,C_{NN}\,\frac{G_V}{\sqrt{2}}\frac{3g_V}{4\pi^2}$$
$$\times D^\dagger_\nu(x)[\bar{p^c}(x)\gamma^5 p(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)],$$
$$\mathcal{L}^{\bar{\nu}_e D\to e^+nn}_{eff}(x) = -\,i\,(1+\bar{\xi})\,g_A\,M_N\,C_{NN}\,\frac{G_V}{\sqrt{2}}\frac{3g_V}{4\pi^2}$$
$$\times D_\nu(x)[\bar{n}(x)\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_e(x)],$$
$$\mathcal{L}^{\nu_e D\to\nu_e np}_{eff}(x) = i\,(1+\bar{\xi})\,g_A\,M_N\,C_{NN}\,\frac{G_F}{\sqrt{2}}\frac{3g_V}{4\pi^2}$$
$$\times D_\nu(x)\,[\bar{p}(x)\gamma^5 n^c(x)]\,[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1-\gamma^5)\psi_{\nu_e}(x)], \quad (A.38)$$

where $\bar{\xi}$ is obtained by using the relation $g_T = \sqrt{3/8}\,g_V$ and is defined by

$$\bar{\xi} = \sqrt{\frac{3}{8}}\,\xi. \quad (A.39)$$

Under the assumption of isotropical invariance of low-energy nuclear forces, the best agreement with the recommended value for the astrophysical factor for the solar proton burning [29] and the contemporary experimental data [36] on the cross-sections for the antineutrino disintegration of the deuteron $\bar{\nu}_e$ + D → e^+ + n + n and $\bar{\nu}_e$ + D → $\bar{\nu}_e$ + n + p caused by charged and neutral weak current, respectively, we obtain at $\bar{\xi} = 0$.

References

1. A. N. Ivanov, H. Oberhummer, N. I. Troitskaya and M. Faber, *The Nambu-Jona-Lasinio model of light nuclei*, Eur. Phys. J. A **7**, 519 (2000).
2. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961).
3. C. E. Rolfs and W. S. Rodney, in *Cauldrons in the Cosmos*, (University of Chicago Press, Chicago London, 1988).
4. A. E. Cox, A. R. Wynchank and C. H. Collie, Nucl. Phys. **74**, 497 (1965).
5. N. Austern, Phys. Rev. **92**, 670 (1953).
6. D. O. Riska and G. E. Brown, Phys. Lett. B **38**, 193 (1972).
7. T.-S. Park, D.-P. Min and M. Rho, Phys. Rev. Lett. **74**, 4153 (1995); Nucl. Phys. A **596**, 515 (1996); T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Phys. Lett. B **472**, 232 (2000).
8. M. J. Savage, K. A. Scaldeferri and M. B. Wise, Nucl. Phys. A **652**, 273 (1999).
9. J.-W. Chen, G. Rupak and M. J. Savage, Nucl. Phys. A **653**, 386 (1999).
10. J.-W. Chen and M. J. Savage, Phys. Rev. C **60**, 065205 (1999); G. Rupak, *Precision Calculation of np → dγ Cross Section for Big-Bang Nucleosynthesis*, nucl-th/9911018, December 1999.
11. S. Weinberg, Phys. Lett. B **251**, 288 (1990); Nucl. Phys. B **363**, 3 (1991); Phys. Lett. B **295**, 114 (1992).
12. D. R. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B **478**, 629 (1996) and references therein; S. R. Beane, T. D. Cohen and D. R. Phillips, Nucl. Phys. A **632**, 445 (1998); T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Nucl. Phys. A **646**, 83 (1999).
13. A. N. Ivanov, M. Nagy and N. I. Troitskaya, Int. J. Mod. Phys. A **7**, 7305 (1992); A. N. Ivanov, Int. J. Mod. Phys. A **8**, 853 (1993); A. N. Ivanov, N. I. Troitskaya and M. Nagy, Int. J. Mod. Phys. A **8**, 2027 (1993); ibid. A **8**, 3425 (1993); Phys. Lett. B **308**, 111 (1993); ibid. B **295**, 308 (1992); A. N. Ivanov and N. I. Troitskaya, Nuovo Cimento A **108**, 555 (1995); Phys. Lett. B **342**, 323 (1995); ibid. B **387**, 386 (1996); Phys. Lett. B **388**, 869 (1996) (Erratum); ibid. B **390**, 341 (1997); F. Hussain, A. N. Ivanov and N. I. Troitskaya, Phys. Lett. B **348**, 609 (1995); ibid. B **369**, 351 (1996).
14. W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
15. L. M. Nath, B. Etemadi and J. D. Kimel, Phys. Rev. D **3**, 2153 (1971).
16. J. Kambor, *The Δ*(1232) *as an Effective Degree of Freedom in Chiral Perturbation Theory*, Talk given at the *Workshop on Chiral Dynamics*, 1997 *Mainz, Germany, September 1–5, 1997*; hep-ph/9711484, November 1997.
17. M. M. Nagels et al., Nucl. Phys. B **147**, 253 (1979).
18. K. Kabir, T. K. Dutta, Muslema Pervin and L. M. Nath, *The Role of Δ*(1232) *in Two-pion Exchange Three-nucleon Potential*, hep-th/9910043, October 1999.
19. A. N. Ivanov, M. Nagy and N. I. Troitskaya, Phys. Rev. C**59**, 451 (1999) and references therein.
20. R. D. Peccei, Phys. Rev. **181**, 1902 (1969) and references therein.
21. M. G. Olsson and E. T. Osypowski, Nucl.Phys. B **87**, 399 (1975).

22. E. Witten, Nucl. Phys. B **160**, 59 (1979).
23. S. P. Klevansky, Rev. Mod. Phys. **64**, (1992) 649 and references therein; T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994) and references therein.
24. K. Kikkawa, Progr. Theor. Phys. **56**, (1976) 947; H. Kleinert, *Proceedings. of International Summer School of Subnuclear Physics*, Erice 1976, Edited by A. Zichichi, p.289. A. Dhar, R. Shankar and S. R. Wadia, Phys. Rev. D **31**, 3256 (1985); D. Ebert and H. Reinhart, Nucl. Phys. B **271**, 188 (1986); M. Wakamatsu, Ann. Phys. (N.Y.) **193**, 287 (1989).
25. J. Bijnens, C. Bruno and E. de Rafael, Nucl. Phys. B **390**, 501 (1993); J. Bijnens, E. de Rafael and H. Zheng, Z. Phys. C **62**, 437 (1994).
26. J. M. Blatt and V. F. Weisskopf, in *Theoretical Nuclear Physics* (John Wiley & Sons, New York Chapman & Hall Ltd, London) 1952.
27. J. N. Bahcall, in *Neutrino Astrophysics* (Cambridge University Press, Cambridge) 1989.
28. J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995); J. N. Bahcall et al., Phys. Rev. **C 54**, 411 (1996).
29. E. G. Adelberger et al., Rev. Mod. Phys. **70**, 1265 (1998).
30. M. Kamionkowski and J. N. Bahcall, ApJ. **420**, 884 (1994).
31. R. Schiavilla et al., Phys. Rev. C **58**, 1263 (1998).
32. T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Astrophys. J. **507**, 443 (1998).
33. X. Kong and F. Ravndal, Nucl. Phys. A **656**, 421 (1999); Phys. Lett. B **470**, 1 (1999); Nucl. Phys. A **665**, 137 (2000).
34. H. Schlattl, A. Bonanno and L. Paterno, Phys. Rev. D **60**, 113002 (1999).
35. J. Boger *et al.*, SNO Collaboration, *Sudbury Neutrino Observatory*, nucl-ex/9910016, October 1999.
36. S. P. Riley, Z. D. Greenwood, W. R. Kroop, L. R. Price, F. Reines, H. W. Sobel, Y. Declais, A. Etenko and M. Skorokhvatov, Phys. Rev. C **59**, (1999) 1780.
37. I. S. Gertsein and R. Jackiw, Phys. Rev. **181**, 1955 (1969).
38. see ref. [26] p.103
39. C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
40. H. Arenhovel and M. Sanzone (editors), *Photodisintegration of the Deuteron: A Review of Theory and Experiment*, (Springer-Verlag, 1991)
41. A. V. Anisovich and V. A. Sadovnikova, Eur. Phys. J. A **2**, 199 (1999).
42. V.V. Anisovich, M.N. Kobrinsky, D.I. Melikhov, A.V. Sarantsev, Nucl. Phys. A **544**, 747 (1992); V.V. Anisovich , D.I. Melikhov, B.C. Metsch and H.R. Petry, Nucl. Phys. A **563**, 549 (1993).
43. K. B. Mather and P. Swan, in *Nuclear Scattering* (Cambridge University Press 1958), pp.212–235.
44. J. R. Bergervoet, P. C. van Campen, W. A. van der Sande and J. J. de Swart, Phys. Rev. **C 38**, 15 (1988).
45. J. N. Bahcall, Astrophys. J. **139**, 318 (1964); J. N. Bahcall and R. M. May, Astrophys. J. **155**, 501 (1969).
46. K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E **3**, 101 (1994).
47. J. N. Bahcall, E. Lisi, D. E. Alburger, L. De Braeckeer, S. J. Freedman and J. Napolitano, Phys. Rev. C **54**, 411 (1996).
48. J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, *SNO: Predictions for Ten Measurable Quantities*, hep-ph/0002293, March 2000.
49. M. L. Goldberger and K. M. Watson, in *Collision Theory*, (John Wiley & Sons, Inc., New York-London-Sydney, 1964).
50. G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. **194**, 1 (1990); G. A. Miller and W. N. T. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, W. C. Haxton and E. M. Henley, eds.,World Scientific Singapore, 1995, p.127.
51. R. Machchleidt and M. K. Banerjee, *Charge-dependence of the* πNN *coupling constant and charge-dependence of the* NN *interaction*, nucl-th/9908066, August 1999.
52. S. L. Mintz, Phys. Rev. C **23**, 421 (1981), ibid. C **24**, 1799 (1981).
53. M. Butler and J.-W. Chen, *Elastic and Inelastic neutrino-Deuteron Scattering in Effective Field Theory*, nucl-th/9905059, June 1999.
54. S. L. Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969); R. Jackiw, in *Lectures on Current Algebra and its Applications* (Princeton University Press, Princeton, New Jersey, 1972); I. S. Gertsein and R. Jackiw, Phys. Rev. **181**, 1955 (1969). R. Jackiw, in *Current Algebra and Anomalies*, edited by S. B. Treiman, R. Jackiw, B. Zumino and E. Witten (World Scientific, Singapore, p.81 and p.211); N. Ogawa, Progr. Theor. Phys. **90**, 717 (1993); R. A. Bertlmann, in *Anomalies in Quantum Field Theory* (Oxford Science Publications, Clarendon Press-Oxford, 1996) pp.227–233 and references therein.
55. J. D. Walecka, Ann. Phys. (NY) **83**, 121 (1974); C. J. Horowitz and B. D. Scrot, Nucl. Phys. A **368**, 503 (1981); Phys. Lett. B **140**, 181 (1984); R. J. Perry, Phys. Lett. B **182**, 269 (1986); T. D. Cohen, Phys. Rev. C **45**, 833 (1992); J. C. Caillon and J. Labarsouque, Phys. Lett. B **311**, 19 (1993); J. Caro, E. Ruiz Arriola and L. L. Salcedo, Phys. Lett. B **383**, 9 (1996); M Matsuzaki, Phys. Rev. C **58**, 3407 (1998).